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The purpose of this introduction is to give a rough overview of the discussion of the formalization of arguments, focusing on deductive arguments. The discussion is structured around four important junctions: i) the notion of *support*, which captures the relation between the conclusion and premises of an argument, ii) the choice of a *formal language* into which the argument is translated in order to make it amenable to evaluation via formal methods, iii) the question of *quality criteria* for such formalizations, and finally iv) the *choice of the underlying logic*. This introductory discussion is supplemented by a brief description of the genesis of the special issue, acknowledgements, and summaries of each article.

1 The Formalization of Arguments

An argument in the philosophical sense is a set of sentences consisting of (at least)[1] one sentence stating a conclusion and (at least) one sentence stating a premise which is or are supposed to support the conclusion.[2] Arguments are of central importance to philosophy not only as a subject of systematic study, but also methodologically as the means to criticise or support philosophical claims and theories. More generally, arguments are an indispensable part of

1 Most arguments discussed by philosophers involve only one conclusion and some have argued against admitting multiple conclusions (see e.g. Steinberger 2011), but there are systematic developments of multiple conclusion logics. See e.g. Shoesmith and Smiley (1978). For the sake of simplicity, I will focus on single conclusion arguments throughout most of this text.

2 Note that throughout this paper I will mostly refer to natural language sentences instead of e.g. utterances of them. I will ignore related metaphysical questions including e.g. questions about what sentences are or about propositions and their relation to natural language sentences and sentences of formal languages. The focus on sentences is both in line with at least significant parts of the literature on formalization and moreover also serves to simplify and homogenize the discussion of different views. I hope that the presentational advantages outweigh the costs of imprecision and a sometimes dangerously liberal use of the term "sentence."

any responsible rational discourse; to give an argument for a claim is to give a reason for it and to set out this reason for oneself and for others to scrutinize.

The analysis, development, and critique of arguments are some of the most important tasks performed by contemporary philosophers working in the analytic tradition. The process of formalization is an important step in any one of these tasks since it makes arguments amenable to the application of formal methods, such as those of model theory or of proof theory. These methods give us precise and objective quality-criteria for arguments, including in particular criteria for their logical validity.

Assuming that we have identified the premises and conclusion of an argument, its formalization will require us to make a number of choices, including those captured by the following four interrelated questions:

1. Which kind of inferential support do the premises lend to the conclusion of the argument?
2. Into which formal language should we translate the argument's premises and conclusion?
3. What makes such a translation into a particular formal language adequate?
4. Which formalisms can be used to evaluate the quality of the argument?

The remainder of this introduction is structured around these four questions about the formalization of arguments. It starts out with a brief discussion of each of these questions in the following four sections, briefly discussing some answers given in the literature and providing some references for further reading. The main aim of this introductory part of this paper is to give readers who are not familiar with the relevant literature a partial look at the more general discussion to which the papers collected in this special issue contribute. This overview is neither comprehensive, nor authoritative. The last two sections of the introduction contain some information about the genesis of the special issue and the editor's acknowledgements and a brief overview of the content of the papers published in this special issue.

2 Inferential Support

A standard classification of arguments individuates kinds of arguments in terms of the kind of *inferential support* which its premises lend to an argument's conclusion. We may accordingly distinguish between, among others, abductive, statistical, inductive, deductive arguments and arguments from analogy. The sort of arguments we encounter in everyday life, e.g. in discussions with neighbours and friends or in political debates, rarely fit into just one of these categories. Rather, they might consist, for example, of an abductive argument for a conclusion which in turn serves as a premise among others in a deductive argument, whose conclusion in turn is used to argue for another claim by analogy, and so on. They may of course also involve particular forms of reasoning which do not neatly fit into the classificatory scheme which one finds in philosophy books, e.g. because they draw on particular non-verbal aspects of a particular discussion, or positively contribute to a debate in a particular context, even though they have the form of a logical fallacy (e.g. an appeal to authority). One might hence argue that theoretical engagement with "real world" arguments require different, perhaps more permissive approaches than those covered in introductory books and courses on logic and critical thinking.[3] Still, many such arguments, or at least parts of them, can be broken down into smaller segments which exemplify one of the canonical argument types.

Deductive arguments enjoy a special status in philosophy due to the particularly strict way in which the premises of a deductive argument supports its conclusion. Consider for example the following argument:

(1) If the train runs late, its passengers will miss their connections.
(2) The train runs late.
(3) ∴ Its passengers will miss their connections.

The conclusion of this argument, which in schemas of this sort will be marked by the prefixed symbol "∴" throughout this text, like that of any valid deductive argument, is logically entailed by its premisses. But what is logical entailment? In contemporary logic, there are two fundamental accounts of what it means for a sentence to be logically entailed by another. The first is the syntactic account which characterizes logical entailment proof-theoretically in terms of derivability or provability in a logical system. Considering the formal language of first-order logic, the core idea of this account is that a sentence s of language is logically entailed by a set of sentences Δ of the same language if, and only if, there is a proof of s which can be constructed in a formal calculus, e.g. using the introduction- and elimination-rules of the logical constants in case of the natural deduction calculus, and taking at most the sentences in Δ as

3 See e.g. Betz (2010, 2013). See also Groarke (2017) for an overview of the field of informal logic.

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hypotheses.[4] The second account is the semantic account, which characterizes entailment in model-theoretic terms. Its core idea is that, focusing again on the language of first order logic, a sentence s (i.e. a well-formed formula of that formal language) is logically entailed by a set of sentences Δ if, and only if, for all models $\mathfrak{M}$ for this language, if all sentences in Δ are true in $\mathfrak{M}$, s is true in $\mathfrak{M}$, where a model is a set-theoretical construction used to semantically interpret all well-formed sentences of the language.[5] As is well-known, the two relations characterized by these accounts coincide for sound and complete logics, such as classical first-order logic, in the sense that they render exactly the same entailments valid. The term "logical consequence" is usually reserved for the latter, semantic notion and I will follow this convention in the remainder of this section.

It is important to distinguish the question of the validity of an argument from that of its *soundness*. An argument is sound if, and only if, it is both valid, i.e. if its conclusion is logically entailed by its premises, and if its premises are true. Neither the proof-theoretic, nor the model-theoretic approach just described is concerned with the truth of an argument's premises. Both approaches target the notion of validity.

The proof-theoretic characterization of deductive entailment is intrinsically linked to particular formal systems which characterize logical expressions like that of negation, conjunction, or the quantifiers in terms of introduction- and elimination-rules which tell us under which conditions we can either introduce or eliminate formulas containing such an expression in the context of a proof. The totality of these rules fix what is provable in such a system and a fortiori give us the sort of syntactic characterization of logical entailment which interests us in the current context. One important philosophical question about introduction- and elimination-rules in a formal system concerns the relation between the two kinds of rules. It was forcefully raised in Prior (1960), who argued against the idea that the meaning of logical expressions is completely fixed by their introduction- and elimination-rules by introducing the connective "tonk" whose associated pair of rules permit us to derive absolutely any sentence from any sentence. An influential idea for how the problem raised by "tonk" and similarly problematic connectives can be avoided is that such connectives violate a harmony-constraint which is

4 The two standard systems in the contemporary discussion (natural deduction and the sequent calculus) were introduced in Gentzen (1935); see Plato (2008);Schröder-Heister (2018) for more general introductions to proof-theory.

5 The key historical text is Tarski (2002); see Beall, Restall, and Sagi (2019) for an introduction.

supposed to govern the relation between a logical expression's introduction- and its elimination-rules.[6] But even if it turned out that such a constraint can be formulated, Prior's argument could still be taken to show that, as Prawitz puts it, "ordinary proof theory has nothing to offer an analysis of logical consequence" (2005, 683).[7] A suitable notion of harmony may give us a way of guarding a formal system against incoherence and a fortiori allow us to accept its harmonious introduction- and elimination-rules as constitutive of the meaning of its logical expressions within that system. Even so, there still would remain an explanatory gap between a formal-system-relative harmonious notion of provability and the general, formal-system-independent notion of logical consequence. One proposal for a way to close this gap is due to Dummett and Prawitz, who argue that logical consequence can be characterized using proof-theoretic means and the notion of canonical proof (see e.g. Dummett 1976; Prawitz 1974, 2005).

Concerning the semantic characterization, many contributors to the recent literature have focused on two different properties which might be used to characterize or define logical consequence, that of being *necessarily truth-preserving* and that of being *formal*.

That logical consequence is closely linked to necessity is a well-established idea in analytic philosophy.[8] In the contemporary debate, this connection is usually spelled out in terms of necessary truth-preservation: If a sentence s is a logical consequence of a set of sentences Γ, then it is necessary that if the sentences in Γ are true, so is s. Or, to put it differently, it is impossible for the sentences in Γ to be true, but for s not to be.

The property of being necessarily truth preserving distinguishes deductive from inductive arguments, such as the following:

(4) Every dog which has been observed up until now likes to chase cats.
(5) Bella is a dog.
(6) ∴ Bella is a dog who likes to chase cats.

6 See e.g. Dummett (1991, ch. 9), and Tennant (1987), Steinberger (2011), and for a recent criticism, Rumfitt (2017).

7 This quote echoes the approach taken by Tarski (1956, 412f), and followed by many contributors to the recent literature, who motivates his semantic definition of logical consequence by arguing against the syntactic approach.

8 See e.g. Wittgenstein's claim that deductive inferences have an inner necessity in §5.1362 of his *Tractatus* (1922).

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Clearly, the fact that every dog observed up until now likes to chase cats does not guarantee that absolutely every dog, including (possibly unobserved) Bella, likes to chase cats. The truth of the premises of this argument, and of those of any inductive argument in general, does not necessitate the truth of its conclusion.[9] The focus of this special issue and of the following parts of this introduction is on deductive arguments.

While necessary truth preservation plausibly gives us a necessary condition for an argument's being deductive, i.e. for its conclusion to be a logical consequence of its premises, there are reasons to doubt that the notion of logical consequence can be adequately explained, characterized, or defined in terms of this property. An important open question in this regard is what kind of necessity the property of necessarily preserving truth involves. The seemingly obvious claim that it is the notion of logical necessity would lead us into an explanatory circle, since logical necessity is plausibly explainable in terms of logical consequence. It is furthermore not clear whether other kinds of necessity, such as for example analyticity, a priority, or metaphysical necessity, can serve this purpose (see Beall, Restall, and Sagi 2019, sec.1).

The second property which is much discussed in the literature on logical consequence is the notion's *formality*. Intuitively speaking, this property distinguishes logical inferences from material entailments such as:

(7) The ball is red.
(8) ∴ The ball is coloured.

Or:

(9) Some dog sees some cat.
(10) ∴ Some cat is seen by some dog.

While these arguments reflect intuitively correct inferences, their conclusions are not logical consequences of their premises. This is because the entailments from (7) to (8) and from (9) to (10) obtain due to the material content of these sentences, i.e. due to what the sentences are about, not due to their form: That (8) is entailed by (7) is guaranteed by the meanings of "is red" and of "is

9 Since both arguments by analogy and statistical arguments can be considered special kinds of inductive arguments (see Salmon 1963, ch. 3), the same holds for them. Abductive arguments also fail to be necessarily truth-preserving, but it can be argued that abduction is not just a special case of induction (see Douven 2017).

coloured" and that (10) is entailed by (9) is guaranteed by the meanings of "sees" and "is seen by."

The validity of a deductive argument in contrast depends solely on the logical form of its premises and conclusion.[10] The logical form of a sentence in turn is determined by the logical expressions it contains and the way they combine with the contained non-logical expressions. That deductive logic is formal in this sense is uncontroversial, but it is hard to say what "formal" means without just defining it ostensively by referring to examples of sentences which we assume to share the same logical form. Can we define the notion of formality in other terms, giving us a systematic criterion to distinguish between the logical and the non-logical expressions of a language? There are several answers to this question two of which will now be briefly introduced.[11] Before this is done, it should be noted that while the focus in the current section is on the notion of logical consequence, most of the discussion of formality focuses on the use of this notion to distinguish logical from non-logical expressions of languages.[12] There is a direct connection between these two loci of formality, since the logical expressions in a sentence determine its logical form and it is in turn the logical form of sentences which ensure that they stand in the relation of logical consequence.[13]

One approach to formality proposed in the literature says that formality can be understood in terms of topic neutrality (see e.g. Ryle 1954, 115ff; Haack 1978, 5–6). The idea is that logical entailments hold irrespective of what the entailed and the entailing sentences are about. What distinguishes the logical expressions of a language is that they, unlike predicates like "is red" and "is coloured" or individual constants, are not about any thing in particular, but that their meaning is rather tied to certain schematic patterns of application which are universally applicable. This criterion for formality gives us a simple and plausible explanation of why the entailment from (7) to (8) is not formal and thus not logical. The main problem noted even by those like Haack who

10 While this clearly holds for the notion of validity one gets e.g. from classical first-order logic, one might see relevance (also: *relevant*) logic as an exception. The core idea of relevance logic is that certain intuitively paradoxical inferences, which are valid in classical logic, can be ruled out as invalid by imposing a relevance constraint to the effect that the conclusion of an argument (or the consequent of a conditional) should not be on a different topic than its premises (the conditional's antecedent). This constraint is however implemented via a formal principle. See Mares (2012) for an overview.

11 For discussions of further answers, see e.g. MacFarlane (2000), Dutilh-Novaes (2011).

12 See e.g. Tarski (1986), Sher (1991), Bonnay (2008).

13 See, however, Sagi (2014) for an alternative view.

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are sympathetic to it is that topic neutrality only gives us a vague criterion for demarcating logical from non-logical expressions: Why could we for example not count the inference from (9) to (10) as formal? After all, it might appear that we can extract a schematic pattern of the following form from this entailment:

(11) x Φs y.
(12) ∴ y is Φed by x.

Putting complications about surface grammar aside which the schema ignores (e.g. "sees" and "is se*en* by"), one may on the one hand argue against its formality by pointing out that the correctness of the inference seems to depend on the seemingly material fact that "Φs" and "is Φed by" are converse relations. On the other hand, one might argue that the two converses are really identical (see Williamson 1985) and then claim that (11) and (12) are just the same sentence in different guises. After stripping away these guises, the inference would really just be a trivial inference from one sentence to itself, instantiating an inference schema which holds irrespective of what the sentence involved means. The point here is of course only that as a criterion for logicality, topic neutrality leaves room for disagreement about particular cases, giving us at best a vague account of what formality is.

The second account of formality is provided by Tarski's classical permutation-invariance-based characterization of logicality (see 1986). This account could be seen as a way to make the topic-neutrality-based account of formality more precise. Its core idea is that the distinguishing feature of logical expressions is that their meaning is invariant under all permutations of the domain of objects of a model. A *model* in the model-theoretic sense is a set-theoretical construction based on a domain of objects which is designed to enable us to semantically interpret sentences of a formal language in set-theoretic terms with respect to that domain. A *permutation of the domain of a model* is a function which maps each object in that domain to a unique object from the same domain. Within a model, first-order predicates can e.g. be interpreted as sets of objects and first-order relational predicates accordingly as sets of tuples of objects. Logical expressions are also given a set-theoretic interpretation, so that first-order quantifiers can e.g. be interpreted in terms of relations between predicates, i.e. sets of tuples of sets of objects. The sets corresponding to material predicates in a model, such as e.g. the relational predicate "is larger than" in a model which is used to interpret a fragment of natural language involving the predicate, vary under at least some permutations of a model's domain. There will e.g. be a permutation which maps two objects *a* and *b* which stand in this relation to other objects from the domain which do not (e.g. simply to *b* and *a*, respectively). The idea underlying Tarski's characterization is that no such thing can happen to logical expressions; the logical expressions retain their intended meaning in a model, no matter under which permutation of the objects in the model's domain we consider them.[14]

One of the main questions about the notion of logical consequence is how the precise, model-theoretic notion relates to the intuitive, pre-theoretical notion of logical entailment with which we operate in ordinary reasoning. The idea that the former can be extracted from natural language, and in particular Glanzberg's recent critique of this idea, are discussed in Gil Sagi's contribution to the special issue.

That there is an explanatory gap to be filled here has already been pointed out by Tarski, who writes that

> the concept of following is not distinguished from other concepts of everyday language by a clearer content or more precisely delineated denotation [...] and one has to reconcile oneself in advance to the fact that every precise definition of the concept [...] will to a greater or lesser degree bear the mark of arbitrariness. (2002, 176)

An influential contribution to the debate about logical consequence which takes this question as its starting point is Etchemendy (1990). Roughly, Etchemendy argues that Tarski's model-theoretic definition of logical consequence fails to capture the intuitive notion of logical consequence, since it presupposes certain contingent, non-logical assumptions about the cardinality of the universe, putting the notion defined by Tarski at odds with the necessity of the intuitive notion.[15]

14 For a more precise explanation of the criterion, see MacFarlane (2015, sec.5) and Bonnay (2014) for an overview of recent work on it. An influential line of objection to invariance-based characterizations of logical constants can for example be traced through Hanson (1997), McCarthy (1981), McGee (1996), Sagi (2015), and Zinke (2018a).

15 See Caret and Hjortland (2015, 5f) and Zinke (2018b, sec.5.3) and see Zinke (2018b, sec.5.1) for a different argument along similar lines.

3 Formal Languages

There are different formal methods which one can apply to evaluate the logical validity of an argument. One may for example rely on semantic methods, such as those provided by a model theoretic semantics, or on syntactical methods, such as the one provided by the natural deduction calculus.[16] In order to apply such formal methods to systematically assess the quality of an argument, the premises and conclusions of arguments have to be translated from the natural language in which they are stated into a suitable formal language. The process of translating a sentence of a natural language into a formal language is the process of formalizing in the narrow sense, as opposed to the wider sense which pertains to whole arguments.

Besides this central technical reason, there are further reasons for formalizing arguments. One important reason is that given a suitable formal language, formalizing an argument forces us to clarify, in different respects, its premises and conclusion. One respect of clarification concerns the many ambiguities present in natural language. Formal languages are often explicitly constructed to be unambiguous, so that each sentence (or formula, if one prefers) of the language is assigned a single, precise meaning. A well-worn example are ambiguous natural language sentences involving quantifier phrases such as "Every child gets a present." Translating the sentence into the formal language of first-order logic, we are forced to decide between two unambiguous readings of the sentence (that every child gets its own present(s) or that every child gets the same present(s)) by the variable-binding structure of the quantifiers of the formal language. Dutilh-Novaes (2012, ch. 4 and 7), furthermore argues that there is another respect in which formalization helps us clarify the formalized parts of language, namely that formal languages serve to eliminate certain cognitive biases.

From the perspective of logic, formal languages are first and foremost mathematical objects.[17] More specifically, they are identified with sets of formulas, where a formula is a sequence of symbols which is generated from a set of symbols, the formal language's alphabet, based on a set of syntactic rules which give us a recipe for generating all well-formed formulas of the respective

16 That logic can help us decide on the validity of an argument formulated in a natural language is a standard assumption. It is however challenged by Baumgartner and Lampert (2008), who argue that the formalization of an argument should rather be understood as a means to explicate the argument by bringing out the formal structure on which the natural language argument is based.

17 But see Dutilh-Novaes (2012, ch. 2) for discussion.

language. The resulting formal language is of course still devoid of meaning, as it merely gives us an alphabet of symbols and rules for constructing certain sequences of them. To interpret the language, a semantics which defines meanings for all well-formed formulas of the language is needed. The standard approach is to identify these meanings with truth-values, reflecting the idea that semantics is about true or false representation of an underlying structure which the sentences of a language reflect or fail to reflect. But there is also an inferentialist tradition which aims to characterize meaning in terms of the inferential rules which govern the expressions of the language.[18]

Formal languages and their semantic interpretations are legion, but what constrains our choice of a formal language when formalizing an argument? This section will focus on one rather important constraint, namely the expressive strength of the formal language. General philosophical constraints about the notions involved in an argument one wants to formalize or pragmatic or sociological constraints tied to certain context will hence not be discussed.

The notion of expressive strength is a semantic notion which concerns not only an uninterpreted formal language, but rather a pairing of such a language with a suitable semantics. It seems that, at least in some cases, there is a notable asymmetry in the relation between the language and the semantics when it comes to determining expressive strength: We cannot extend the expressive strength of some language beyond a certain threshold set by the expressions it contains by coupling it with a different semantics. An example is the language of propositional logic which simply lacks the syntactic expressions needed to capture the inner logical structure of atomic formulas which grounds the felicity of certain inferences which come out as valid in classical first-order logic. One could try to compensate for the lack of syntactic structure by adopting a particular translation scheme and by encoding the validity of the logically invalid inferences in the semantics. E.g. if the predicate “F” stands for “is a dog” and “G” for “is an animal,” then the valid first-order inference from “$\forall x(Fx \rightarrow Gx)$” and “$\exists xFx$” to “$\exists xGx$” could be simulated in the language of propositional logic by assigning a propositional constant to the English sentences “All dogs are animals,” “There is a dog,” and “There is an animal” and by building it into one’s semantics of the language of propositional logic that the two first entail the third. But there are obvious limits to this strategy, since it e.g. makes the semantics depend on a particular translation-schema from a natural into the formal language and since it would make it a matter

18 See e.g. Sellars (1953), Brandom (1994), Peregrin (2014).

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of stipulation which propositional constants express logical truths or stand in relations of logical entailment.

In order to allow us to adequately formalize an argument, the formal language (together with a suitable semantic interpretation), has to be able to capture enough of the logical structure of the argument as stated in a natural language to make it an argument, i.e. a collection of sentences one of which stands in a relation of inferential support to the others. Intensional logic offers a wealth of examples which highlight expressive limitations of certain formal languages. A classical example from tense logic concerns the formalization of the sentence (see e.g. Cresswell 1990, 18):

(13) One day all persons now alive will be dead.

In the language of a simple tense logic which extends the language of first-order logic with the sentential tense-operators **P** ("It was the case that...") and **F** ("It will be the case that..."), if one uses the predicates A, D for "... is alive" and "... is dead" respectively, the closest one can get to an adequate formalization of (13) is:

(14) $\mathbf{F}\forall x(Ax \rightarrow Dx)$

Since this formula says that it will be the case at a future time that everyone alive at that time is dead at that time, this translation is clearly inadequate. There are different ways to remedy this lack of expressive strength. One is to add a sentential "now"-operator **N** and to introduce a double-indexed semantics for the language which allows one to evaluate formulas relative to not one but two time indices, one of which is specifies the time of evaluation.[19] Figuratively speaking, **N**'s semantic contribution to a formula is to force the evaluation of the formula in its scope at the time of evaluation. So in

(15) $\mathbf{F}\forall x(\mathbf{N}Ax \rightarrow Dx)$

N's job is to exempt the atomic formula Ax from being evaluated at the future time index introduced by **F** and to force its evaluation at the time index representing the time of evaluation, i.e. present time from the perspective of someone evaluating the formula. The result is an adequate formalization of (13) which could e.g. be used in the formalization an argument involving (15) as a premise.

19 See e.g. Vlach (1973), Kamp (1971).

Interestingly, (13) can also be expressed without temporal operators, if we instead allow the quantifiers of the language to range over times, relativize predications to times, so that "*Axt*" and "*Dxt*" stand for "*x* is alive at time *t*" and "*x* is dead at time *t*" respectively, and take t_0 to stand for the time of evaluation (Cresswell 1990, 19):

(16) $\exists t_1(t_0 < t_1 \wedge \forall x(A(xt_0) \rightarrow D(xt_1)))$

This formula seems to adequately capture what (13) says relative to a particular time of evaluation. Note that, as Cresswell (1990, 21) points out, it might be argued to be objectionable that (16) produces an eternal sentence for each value of t_0. At least it is, if we assume that the truth-value of (13) could change, if e.g. technological advances would allow humans to attain immortality.

The availability of (16) as a translation of (13) raises the question of whether it wouldn't be preferable to just work with the language of first-order logic rather than with the extended language of first-order tense logic which adds new operators. Considerations of parsimony certainly seem to favour this strategy. Why introduce additional operators if we can express the same things without them? Philosophical reasons may be brought to bear on this question. Arthur Prior for example argued that the tense logical formalization of (13) is preferable, considerations of parsimony notwithstanding, since he took tense, which is more naturally expressed using operators like **F**, **P**, and **N**, to be more fundamental than time.[20]

Questions about the choice of formal language are discussed in Hanoch Ben-Yami and, with a historical focus on Frege's *Begriffsschrift*, in Jongool Kim's contributions to the special issue.

4 Quality-Criteria for Formalization

4.1 *Translation Problems and a Simple Quality Constraint*

Assuming that a suitable formal language has been selected, determining the logical form of a natural language sentence is still not a straightforward matter. It seems clear that not every formula of such a language can equally well be used to translate every natural language sentence. But what then makes a

20 See Cresswell (1990, 22) and see Lewis (1968) for the development of counterpart theory, a theory expressible in the language of first-order logic which can express any sentence which can be expressed in the language of first-order modal logic.

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formula or a set of formulas an adequate or a correct formalization? Can we formulate general criteria for the quality or admissibility for formalizations of a formal language?[21]

A minimal constraint on the correctness of formalization of sentences is that it should respect certain intuitively valid inferences involving these sentences. In this subsection, the focus will be on two well-known examples of problem cases for translations of natural language sentences into the language of first-order logic which illustrate two different attempts to ensure that this minimal constraint is met.

The first problem specifically concerns a particular type of sentence, namely that of action sentences. Consider the following sentence:

(17) Donald embraced Orman at noon.

The most-straightforward translation of this sentence into the language of first-order logic is

(18) $Edon$

where $Exyz$ is the three-place predicate "x embraces y at time z" and d, o, n are individual constants designating Donald, Orman, and the relevant point in time respectively. The problem with this formalization of the sentence is that it does not respect the inferential relation between (17) and the following sentence:

(19) Donald embraced Orman.

Clearly, if Donald embraced Orman at noon, Donald embraced Orman. Yet, if we translate (19) in the same straightforward manner as (17), using a two-place predicate Fxy which stands for a sentence of the form "x embraces y," we get the following formula:

(20) Fdo

But this formula is not logically entailed, in classical first-order logic, by (18). A classic discussion of this problem is found in Davidson (1967). Building on previous work by Reichenbach and Kenny, Davidson's solution to the problem

21 This is a topic which has surprisingly not been discussed much in the literature. Adequacy criteria for formalizations in first-order logic are for example discussed in Baumgartner and Lampert (2008); Baumgartner (2010), Blau (1977), Brun (2004, 2012), Epstein (1994), and Sainsbury (2001).

is to propose an alternative formalization-pattern for sequences describing events. According to his proposal, (17) should be formalized as:

(21) $\exists x(Gxdo \wedge Hxn)$,

Here the predicate $Gxyz$ stands for "x is an embrace by y of z," the predicate Hxy for "x happened at time y," and the constants d, o, n retain their earlier referents. This new formula directly entails the formula

(22) $\exists x Gxdo$

which, following Davidson's formalization pattern, is an adequate formalization of (19). The problem is hence solved.

Davidson's proposal gives us an example of a formalization pattern which is sensitive to the content of the formalized sentence. As Davidson put it: "Part of what we must learn when we learn the meaning of any predicate is how many places it has, and what sorts of entities the variables that hold these places range over. Some predicates have an event-place, some do not" (1967, 93). Given the previous discussion about the distinction between formal and material inferences, one might think that Davidson's proposal blurs the line between the two kinds of inferences, if such a line can at all be drawn. One might indeed think that both the example discussed by Davidson and the example to be discussed next illustrate that it is, even in the case of first-order logic, a genuinely open question to which extent formal logic can account for the informal notion of entailment, including ostensibly material entailments such as those from (7) to (8) and from (9) to (10).

The second example illustrates a problem case of formalization which arises even if one accepts external constraints on formalization. A classical example discussed in the literature is De Morgan's problem:[22]

(23) All horses are animals.
(24) ∴ All heads of horses are heads of animals.

There is a straightforward way to formalize (23) by simply translating "is a horse" using the predicate-letter F and "is an animal" using the predicate letter G:

(25) $\forall x(Fx \rightarrow Gx)$

22 See Brun (2004, sect. 9, 189ff). See also Brun (2012).

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If we formalize (24) in the same manner using the predicate-letter H for "is a head of a horse" and I for "is the head of an animal," we end up with:

(26) $\forall x(Hx \rightarrow Ix)$

If we just consider (24) in isolation, this is may be a fine formalization, but (26) is inadequate in the context of a formalization of the argument from (23) to (24). The inference captured in this argument is intuitively correct, but (25) does not logically entail (26) .

There are different formalizations of (24) which solve the problem (cf. Brun 2004, 193). One solution is to formalize (24) as follows, using the binary predicate K to translate "is the head of" in addition to F and G which are still used to translate "is a horse" and "is an animal" respectively:

(27) $\forall x \forall y((Fy \wedge Kxy) \rightarrow (Gy \wedge Kxy))$

Alternatively, the following formula also does the trick:

(28) $\forall x(\exists y(Fy \wedge Kxy) \rightarrow \exists y(Gy \wedge Kxy))$

Both (27) and (28) are logical consequences of (25), so both (25) and (27), as well as (25) and (28) give us formalizations of the argument from (23) to (24) which can be said to meet the minimal requirement set out earlier in this section. Interestingly however, (27) is logically stronger than (28) in the sense that (28) is a logical consequence of (27), but (27) not of (28). The fact that we can have two different, but non-equivalent ways of formalizing the argument from (23) to (24) raises several general questions about the formalization of arguments (cf. Brun 2004, 194). We might for example ask whether the two variants can be compared concerning their quality as formalizations of the natural language argument they translate, and if so, which one of them offers us the better formalization.

The discussion of the two classical formalization problems illustrate two important general aspect of how we determine the correctness of a formalization. The first and quite obvious point is that the intuitive notion of inference we apply when reasoning using natural language gives us a corrective for correct formalization. The correctness of a formalization can never be a completely formal matter; i.e. logic alone can never tell us whether a formula is a correct formalization of a sentence.[23] Second, whether a formula of a formal

23 Which is of course not to say that we cannot use formal methods to reason about correctness, see Paseau (2019).

language is an adequate formalization of a natural language sentence cannot be determined by considering the sentence in isolation. Correctness rather is a holistic notion which has to take relevant inferential patterns in natural language into account. (Cf. Friedrich Reinmuth's contribution to this special issue.)

These two points give us constraints on adequate formalization, but they obviously fall short of giving us general criteria for the adequateness of formalizations which might, e.g. answer the mentioned questions about the comparative quality of equally admissible alternative formalizations.

4.2 *General Quality Criteria*

What shape could such a general criterion take? Brun distinguishes two kinds of quality criteria, *correctness criteria* and *adequacy criteria* (see 2004, 11). In his terminology, a formalization is *correct* if its validity-relevant features are just those of the sentence or of the argument which it formalizes. But there is a fundamental problem for formalizing arguments which shows that correctness alone is not enough to guarantee that a formalization is a good formalization. Following Blau (1977), this problem has come to be known as the problem of unscrupulous formalization.[24] To see the problem, consider the following example given in Brun (2004, 238):

(29) Every prime number is odd or equal to 2.
(30) There is no prime number which is not odd and not equal to 2.

These two sentences can arguably be recognized to say the same without thinking much about their logical form, e.g. by pondering the meanings of "every" and "there is no." Let us, for the sake of the argument, assume that we accept on an intuitive level that (29) and (30) are equivalent. Using "P" for "is a prime number" and "O" for "is an odd number," a scrupulous formalization of the two sentences would give us the two following formulas:

(31) $\forall x(Px \rightarrow (Ox \vee x = 2))$
(32) $\neg\exists x(Px \wedge (\neg Ox \wedge \neg x = 2))$

Given these translations, we could now provide a formal explanation of our informal judgement that (29) and (30) are equivalent by proving that the two formulas are equivalent in first-order logic. An unscrupulous formalization in

24 Blau's German term is "skrupellose Formalisierung" (see 1977, 18).

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contrast would for example be one which translates both (29) and (30) as (31). The goal of our exercise in formalization is to show that we can confirm our informal judgement that (29) and (30) are equivalent and there is no easier equivalence proof than one which demonstrates that a formula, trivially, but correctly, is equivalent to itself. The point of the example is that if correctness is all that matters, then there the unscrupulous formalization is as good as the scrupulous one.

The example of unscrupulous formalizations shows that correctness alone is not a guarantee of the quality of a formalization. This is where adequacy enters the picture. Adequacy is a stricter quality-criterion than correctness, that is, each adequate formalization is a correct formalization, but not vice versa. The notion of adequacy hence allows us to rule out correct, but still problematic formalizations of the sort just discussed. Unscrupulous formalization give us a clear adequacy-constraint: Adequate formalizations do not trivialize non-trivial inferential connections between the resulting formulas, ruling out e.g. a formalization which translates both (29) and (30) as (31). Accordingly, adequacy criteria go beyond correctness criteria in the sense that they ensure that the formalization not only captures the validity-relevant features of the formalized sentences or argument, but also does so in a non-trivial way.

There are, just as in case of the notion of logical entailment, two different conceptions of correctness which are tied to two conceptions of what validity-relevant features are. First, these features can be the truth-conditions of the relevant sentences and formulas, giving us a semantic conception of correctness. The idea then is that a formalization is correct if the formalization has the same truth-conditions as the sentence it formalizes relative to a logic and a translation-schema (or correspondence schema in Brun's terms) which specifies the translations of all relevant expressions of natural language into the relevant formal language.[25]

The validity-relevant features can however also be inferential features, giving us a syntactic conception of correctness. For arguments, the formalization and the formalized argument as stated in natural language have to have the same inferential structure, whereas for the formalization of a single sentence, the formalization is correct if the formally correct inferences in which it can occur are also valid in an informal sense for the corresponding inferences made in natural language.[26]

25 See the correctness principle (WK) in Brun (2004, 210).
26 See the correctness principle (SK) in Brun (2004, 214).

The minimal constraint mentioned in the previous subsection hence concerns the second, the inferential, notion of correctness. Sainsbury discusses the following adequacy criterion for formalizations of English sentences:

> QC1. A formalization is adequate only if each of its logical constants is matched by a single English expression making the same contribution to truth conditions. (Sainsbury 2001, 352)

This proposal is motivated by Sainsbury's discussion of what he calls the "Tractarian vision," that every entailment is a logical entailment. Friends of this idea might be tempted to ensure that material entailments are really logical entailments by putting more structure into the formalizations than the surface form of the sentences requires. They might for example try to ensure that the argument from (7) ("The ball is red") to (8) ("The ball is coloured") counts as logically valid by formalizing its premise and conclusion as follows:

(33) $Rb \wedge Cb$
(34) Cb

A problem with this sort of translation and, more generally, with the Tractarian vision is that it appears to conflate the two distinct projects of analysing the meaning of a sentence and of isolating its logical form.[27] The motivation for formalizing (7) as (33) has to draw on the semantic fact that to say that an object is red is, implicitly, to say that it is coloured. To ensure that the entailment is logical, the proposed formalization hence draws on a fact about the meaning of the non-logical expressions involved in (7). So while the formalization of the argument works on the formal level, it indirectly violates the formality requirement: The formality of the logical entailment between (33) and (34) is not mirrored by the premise and conclusion of the argument as stated in English. Sainsbury's adequacy criterion QC1 systematically blocks ad hoc logicalizations of arguments of this sort.[28]

A drawback of QC1 is that it also threatens Davidson's proposed formalization schema for action sequences: There is arguably no single English

27 See Sainsbury (2001, 354). Note that such translations would also count as unscrupulous in Blau's and Brun's sense.

28 Note that this problem would not arise in the first place in a logically perfect language of the sort which Wittgenstein characterizes in the *Tractatus*. In such a language, all logically simple sentences are fully analyzed in the sense that they do not contain any hidden logical or semantic structure which could be brought out by formalizing them.

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expression in “Donald embraced Orman at noon” which makes the same contribution to the sentences’s truth conditions as the existential quantifier in its formalization (21) does with respect to that formula of first-order logic.

Purists who eschew the content sensitivity of Davidson’s formalization pattern might see this as an advantage rather than a drawback, but Brun argues that QC1 suffers from two further problems which are less specific and more severe (see Brun 2004, 253f). First, it presupposes an explanation of what it means for a natural language expression to match or correspond to a logical constant in a formula of the formal language into which one translates. Second, putting the first problem aside, while QC1 rules out some problematic formalizations, such as (33), it likewise rules out uncontroversial formalizations, including in particular:

(35) Müller is sad, Schmidt is happy.
(36) $Sm \wedge Hs$
(37) Crocodiles are green.
(38) $\forall x(Cx \rightarrow Gx)$
(39) Hans owns a red bicycle.
(40) $\exists x(Bx \wedge Rx \wedge Ohx)$

The comma in (35) can hardly be said to make the same contribution to its truth-conditions as the conjunction in (36) and the same can be said about the quantifier and the material conditional in (38) and the existential quantifier, as well as the two conjunctions in (40). QC1 helps rooting out some inadequate formalizations, but it throws the baby out with the bathwater by classifying a range of standard formalizations as inadequate.

There are however better adequacy criteria than QC1, such as the following, (a simplified version of) Brun’s criterion of less precise formalization which gives us a necessary condition for the adequacy of a formalization:

> QC2. For a formula ϕ to be a correct formalization of a sentence A, every formula ψ which is less precise than ϕ has to be such that there is a correct formalization of A which is a notational variant of ψ.[29]

29 Cf. principle (UGK), Brun (2004, 349).

This principle needs a bit of unpacking.[30] First of all, "less precise" is here understood to be a relation which holds between two formulas ϕ and ψ relative to a formalism (i.e. a logic), which are formalizations of the same sentence and which are such that ψ can be generated from ϕ by substituting a logically more complex formula for a sub-formula of ϕ. Of two such formulas, one is less precise than the other if the former gives us a less detailed picture of the logical structure of the sentence. Consider for example the following sentence:

(41) Paul Otto Alfred is an adopted son.

Letting the constant a stand for the name "Paul Otto Alfred" and the predicate P for "is an adopted son," we can formalize (41) as:

(42) Pa

However, we could also use the two predicates Q and R, standing for "is adopted" and "is a son" to formalize (41) as:

(43) $Qa \wedge Ra$

Or we could still be more precise and formalize (41) as follows using the predicate S to translate "is male" and T to translate "is the father of":

(44) $Qa \wedge Sa \wedge \exists x(Txa)$

(42)–(44) are all formalizations of the same sentence, namely (41); furthermore, each of the three formulas can be generated by substitution from the others;[31] finally, the three formulas are increasingly precise, revealing more and more of the formalized sentence's logical structure.

QC2 also involves the notion of a notational variant. This notion can be understood in terms of substitution: A formula ϕ is a notational variant of a formula ψ if, and only if, ϕ can be transformed into ψ by a one-to-one substitution of non-logical predicates and vice versa (see Brun (2004), 301).

Now how does QC2 work? We can think of a logically complex formalization as the result of a step-by-step procedure which starts with an atomic formula and then begins capturing more of the formalized sentence's logical structure

30 Just as with the principle itself, I will in the following simplify the details of Brun's account which is explained in full detail in (2004, sec.13.2 and 13.4).

31 E.g. we get (43) from (42) by substituting Pa by $Qa \wedge Ra$ and (44) from (43) by substituting $Sa \wedge \exists x(Txa)$ for Ra.

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by analyzing it in terms of more complex formulas which all are correct in the semantic sense of having the right truth-conditions. What QC2 tells us is basically that to be an adequate formalization is to only contain logical complexity which can be the result of such a process of refinement. (44) for example counts as adequate in this sense, since if we condense the second conjunction into a single formula, we in any case get a formula which is a notational variant of (43), and which is a semantically correct formalization of the sentence.

With that said, let us return to De Morgan's problem and the two non-equivalent, but seemingly both admissible formalizations of (24), (27) and (28):

(27) $\forall x \forall y((Fy \wedge Kxy) \rightarrow (Gy \wedge Kxy))$
(28) $\forall x(\exists y(Fy \wedge Kxy) \rightarrow \exists y(Gy \wedge Kxy))$

Can QC2 help us decide whether one of the two is a more adequate formalization of (24), the conclusion of De Morgan's argument? Note first that neither of these two formulas is more precise than the other in the relevant sense, since the quantifiers and variables the two formulas contain prevent us from generating one from the other by substituting a logically more complex formula for a sub-formula in either of the two. However, only one of the two formulas, namely (28) stands in the "is more precise than"-relation to (26):

(26) $\forall x(Hx \rightarrow Ix)$

We can generate (28) from (26) by substituting $\exists y(Fy \wedge Kxy)$ and $\exists y(Gy \wedge Kxy)$ for Hx and Ix respectively. (27) cannot be generated in the same way, since the second universal quantifier in (27) cannot be introduced by substituting logically more complex formulas for sub-formulas of (26). The closest we can get to (26) is:

(45) $\forall x \forall y(Mxy \rightarrow Nxy)$

However, it is not clear what the predicates *M* and *N* could stand for. Since both are relational predicates, *M* would have to correspond to something like "is a horse head of" and *N* to "is an animal head of." Be that as it may, since (45) is a less precise formula than (27), QC2 tells us that (27) is an inadequate formalization of (24), unless there is a notational variant of (45) which is an adequate formalization of (24) ("All heads of horses are heads of animals"). If (45) turned out to be a notational variant of (26), then this condition would be met. However, this is not the case, since due to the presence of the second universal quantifier in (27), we cannot generate it from (26) by one-for-one substituting its non-logical predicates. So whether (27) is an adequate formalization of (24) depends on whether (45) is an adequate formalization of (24).

This opens up a way to informally argue that only (28) is an adequate formalization of (24) by arguing that (45) is not a notational variant of an adequate formalization of (24). Given QC2, the adequacy of (45) cannot be justified by pointing out that it is a less precise formula than the adequate formalization (27) since it is exactly the adequacy of (27) which is at issue, so an independent justification is needed. One might then for example argue that the additional logical complexity of (45) gives us a reason to prefer (26) instead, or one might also target the seemingly unnatural translation schema one would have to adopt to make sense of (45).[32]

5 Choice of Logic

Since our focus here is on deductive logic, the formalisms one has to choose from when formalizing an argument are different logics. The one logic which has the claim to being the default choice is classical first-order logic. It has this status in virtue of some of its formal properties—classical first-order logic is e.g. complete and sound—and its expressive strength. First-order logic can be used to formalize a range of mathematical theories, including e.g. some set theories and, as we have seen, it can be used to express the same, or at least similar claims, as intensional logics such as tense logic or modal logic (see Lewis 1968).

Still, there appear to be reasons to rely on alternative logics. One reason is that one may be compelled to reject logical principles or inference schemata which hold in e.g. classical first-order logic with respect to certain contexts, or topics, or more generally for philosophical reasons. Free logic provides an example of the latter sort. As Karel Lambert describes it, free logic is "free of existence assumptions with respect to its terms, general and singular" (1981, 123). Classical first-order logic involves the assumption that every singular term (e.g. each constant) refers to an object in the domain of quantification.[33] This, free logicians argue, is problematic. Consider for example the sentence:

32 Note that Brun uses an additional adequacy criterion to more formally argue that (28), and not (27), is an adequate formalization of (24) (see -Brun (2004, 352–56).

33 See e.g. Frege (1893, I:9, note 31).

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(46) Heimdallr exists.

In the language of first-order logic, this sentence can be formalized as follows, using the constant h for Heimdallr:

(47) $\exists x(h = x)$

Literally, this formula says that there exists something the same as Heimdallr. Both this logico-literal restatement and (46) itself are, at least insofar as common sense is concerned, false, since Heimdallr is an object of fiction, i.e. an object which does not exist. Given the mentioned assumption about the reference of singular terms, this formula is however a logical truth of classical first-order logic. If we accept first-order logic, we hence seem to be forced to accept an obvious falsehood as true.[34] Free logic offers a way out of this problem, since it allows for the falsity of formulas like (47). This is because unlike in classical logic, the rule of Existential Generalization:

(48) $A \vdash \exists x A(x/t)$

fails in free logic. Here, A is a formula of the language of first order logic and $A(x/t)$ is the formula which results if we replace any occurrence of the individual constant t by the variable x (if there are any). Existential Generalization allows us to e.g. infer from (the formalization in the language of first-order predicate logic of) "Heidallr owns Gjallarhorn" to the existence of something which owns Gjallarhorn. In free logic, this inference is not valid, since, briefly put, that a sentence is satisfied by a particular individual constant does not entail the existence of an object in the domain of discourse which satisfies the formula.[35] Other reasons for adopting particular (non-classical) logics which have been given in the philosophical literature include its adequacy for explaining vagueness (cf. e.g. Machina (1976) or Smith (2008)), or the need to move to a non-classical logic in order to avoid semantic paradoxes such as the liar paradox (cf. e.g. Kripke 1975).

It is a fact that there are different logics, but which one should we rely on in analyzing arguments? Carnap famously adopted a tolerant stance towards logic. He assumed that any choice of logic is permissible in principle and that

34 There are ways to evade this argument, e.g. by adopting the descriptivist theory of proper names famously proposed in B. A. W. Russell (1905). The dominant view about the reference of proper names, according to which they are directly referential (cf. Kripke 1980), however, speaks against Russell's theory.

35 See Nolt (2014) for a general overview and further explanation.

which logic one relies on is ultimately a matter of its usefulness for a particular purpose.[36] However, Carnap's tolerant attitude is not shared by everyone and we may ask whether, despite the fact that there are different logics, there is one logic which is correct in the sense that it gives us the one correct notion of logical consequence. This question is asked in the recent discussion about logical pluralism, the view that there is more than one correct logic and therefore also more than one correct notion of logical consequence.[37] A recently proposed methodology for choosing between logics based on reflective equilibrium is criticized in Bogdan Dicher's contribution to the special issue. A question about the independence of formalization and choice of logic is raised in Roy Cook's contribution.

6 Genesis of the Special Issue and Acknowledgements

The initial idea for this special issue came about during the workshop "Making it (too) precise" which I organized together with Dominik Aeschbacher and Maria Scarpati in July 2017 at the University of Geneva as part of the SNSF-funded research project "Indeterminacy and Formal Concepts" (project nr. 156554) led by Prof. Kevin Mulligan. After the editorial committee of *Dialectica* approved the proposal for the special issue, an open call for papers was published online. 18 papers in total were submitted, including some of those presented at the workshop in Geneva. All of these paper were subject to the same review process which mirrored that passed by regular submissions to *dialectica*, with the sole differences being that the guest editor was both responsible for the organization of the review process and for the initial internal review. The 13 papers which passed this initial step were double-anonymously reviewed by two expert reviewers. In a third and final step, the papers which were selected by the guest editor based on the recommendations of the reviewers were presented to the editorial committee and the editors who approved the guest editor's decision.

First and foremost, I would like to thank the authors for contributing their papers and allowing them to be published in this special issue. My second greatest debt is to all the reviewers whose work made it possible for an interested bystander like myself to take editorial decisions. I would also like to thank the editorial committee of *Dialectica*, especially Matthias Egg for his

36 See in particular Carnap's principle of tolerance, as set out e.g. in Carnap (1947, sec.17).

37 See Beall and Restall (2006) and Shapiro (2014) for developments of the position, Field (2009), Priest (2006), Read (2006) for opposing views, and G. Russell (2019) for an overview.

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helpful comments and its managing editor Philipp Blum, for giving me the opportunity to edit and for approving the special issue and the Swiss National Science Foundation for financial support at the outset ("Indeterminacy and Formal Concepts," University of Geneva 2014–17, project number 156554, PI: Kevin Mulligan). Finally, I would like to thank Philipp Blum and all the people involved for the work they put into turning *Dialectica* into an open access journal. It is a very happy coincidence, one which only materialized after the reviewing process had been well under way, that this special issue would be one of the first issues of the journal to be freely and openly accessible to anyone over the internet.

7 Overview of the Papers of the Special Issue

In his paper "The Quantified Argument Calculus and Natural Logic," Hanoch Ben-Yami relates his Quantified Argument Calculus (acronym: *Quarc*) to Larry Moss's Natural Logic. The main selling point of both of these logical systems is that they give us logics which are able to account for the validity of certain intuitively correct argument types, such as for example the argument from (7) to (8), which are invalid in classical first-order logic. Ben-Yami shows that Quarc is able to account for the same extended range of arguments which Moss's Natural Logic is designed to capture and furthermore argues that Quarc has the advantage that it does not require to posit negative nouns to do so.

In "Reflective Equilibrium on the Fringe: The Tragic Threefold Story of a Failed Methodology for Logical Theorising," Bogdan Dicher critizises the idea due to Peregrin and Svoboda (2017) that reflective equilibrium can serve as a method for choosing a logic. The core idea of this approach is that the fact that the rules of inference of a logic and the inferences in natural language which it is supposed to formalize can be brought into a (virtuously circular) agreement with each other provides us with a criterion for that logic's adequacy. Dicher's argument against this idea is based on three case studies, one focusing on the impact on harmony of moving from single- to multiple-conclusion, another focusing on the question of how we may distinguish between logics which deliver the same valid logical entailments, focusing on classical first-order logic and strict-tolerant logic (Cobreros et al. 2012), and a third focusing on an application of the logic of first-degree entailment (Anderson and Belnap 1975) by Beall.

Jongool Kim's paper "The Primacy of the Universal Quantifier in Frege's Concept-Script" focuses on the question of why Frege adopted the universal, rather than the existential quantifier as a primitive of the formal system developed in his Frege (1879). This question is not only of historical interest, given that Frege's book is one of the most important contributions to the development of contemporary logic, but also raises a general systematic question about factors motivating the choice of a particular formal language. While Frege never explicitly answered this question, Kim extracts, develops, and discusses three arguments which support this choice from Frege's works and singles out one of them, a philosophical argument based on the idea that choosing the existential quantifier as a primitive instead would have undermined Frege's logicist project of putting arithmetic on a purely logical foundation, as the strongest.

Friedrich Reinmuth's paper "Holistic Inferential Criteria of Adequate Formalization" focuses on adequacy criteria for logical formalization. Following e.g. Brun (2004), Peregrin and Svoboda (2017) and others, Reinmuth assumes that such criteria have to be holistic in the sense that they have to take into account the consequences of the choice one makes in formalizing a particular natural language sentence not only for the target argument, but also for all other arguments involving the same sentence as a premise or conclusion. He points out shortcomings in existing proposals and motivates and develops criteria which extend from arguments to more complex sequences of logical reasoning and which e.g. allow one to distinguish between equivalent formalizations of arguments which nonetheless lead to differences when embedded in such sequences.

Gil Sagi's paper "Considerations on Logical Consequence and Natural Language" focuses on the relation between the notion of logical consequence and ordinary language. Sagi in particular targets three recent arguments due to Glanzberg (2015) to the conclusion that the relation of logical consequence cannot be simply read off natural language. Her paper rebuts these arguments and argues that one of the two positive proposals made by Glanzberg for how one might go beyond natural language in order to get at logical consequence is in fact compatible with the view that this relation exists in natural language.

In " 'Unless' is 'Or,' Unless '$\neg A$ Unless A' is Invalid," Roy T. Cook discusses the formalization of arguments involving the expression "unless," focussing in particular on the differences between formalizations which rely on the same formal language, that of propositional logic, but differ in that they assume classical or intuitionistic logic as the background logic. One of Cook's main

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points is that his discussion questions the assumption that translations from informal into formal language are logic neutral, in the sense that we can settle for a logical formalization independently of first adopting a particular logic.

Vladan Djordjevic's paper "Assumptions, Hypotheses, and Antecedents" focuses on an important distinction between three ways in which deductive arguments can be cast both in formal languages and in natural language. Djordjevic distinguishes "arguments from assumptions," which are arguments in which each premise is assumed to be logically true and the logical truth of the conclusion is to be established, from "arguments from hypotheses," in which the validity of an inference from the premises to the conclusion is at issue, and from assertions of conditionals which contain the premises of an argument in their antecedent and its conclusion in its consequent. The three categories are often conflated and Djordjevic argues that certain philosophical puzzles, including a standard argument for fatalism and McGee's counterexample to Modus Ponens can be resolved based on these distinctions.

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