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The Tragic Threefold Story of a Failed Methodology for Logical Theorising

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Reflective equilibrium, as a methodology for the "formation of logics," fails on the *fringe*, where intricate details can make or break a logical theory. On the fringe, the process of theorification cannot be methodologically governed by anything like reflective equilibrium. When logical theorising gets tricky, there is nothing on the pre-theoretical side on which our theoretical claims can reflect of—at least not in any meaningful way. Indeed, the fringe is exclusively the domain of theoretical negotiations and the methodological power of reflective equilibrium is merely nominal.

Reflective equilibrium has been proposed as a methodology for logical theorising and, indeed, as a procedure for justifying our logical knowledge at least since Goodman's "new riddle of induction."[1]

In recent years, interest in it resurged, particularly in the wake of the advances of the *anti-exceptionalist* programme in logic. The general background for this paper will be given by a modest form of anti-exceptionalism, compatible with logical immanentism—the view that logic is immanent in language (see e.g. Brandom 2000)—which claims that the epistemology of logics is fallibilist (see e.g. Peregrin and Svoboda 2013, 2016, 2017; Read 2000).[2]

In this paper, I will argue against the thesis that reflective equilibrium is a viable methodology for logical theorising. This negative thesis does not deny that the *phenomenology* of logical inquiry could be described, at least in part, in accordance to the pattern provided by reflective equilibrium (hereafter often abbreviated as "*RE*"). This I gladly grant and duly deplore, for I believe

1 In Goodman (1955). The name, of course, is of a later date, being first used in Rawls (1971).

2 Full-blooded anti-exceptionalism is, roughly, the view that logic is not special, but rather contiguous with the empirical sciences (Hjortland 2017; Priest 2014; Russell 2014; Williamson 2007).

that, ultimately, it is the plausibility of this way of describing logical inquiry that is at the core of the misguided tenet that *RE* is a meaningful methodology for logic. Instead, my claim is that the processes normally associated with logical investigations are too complex, too abstract, and too "theoretical" to be in any *substantive* sense guided by *RE*. I will present my arguments against reflective equilibrium via three case studies of currently debated issues among logicians. These vignettes will, I hope, drive home the following three points:

- The first is that logical theorising is systematically biased in favour of theoretical considerations and so *RE* is, *qua* methodology, too weak.
- The second is that *RE* underdetermines both the identification of the specific problems one encounters in "the formation of logics", i.e. problematisation, and the problem-solving process itself.
- The third and final point I wish to make is that *RE* systematically favours weaker logics.

1 Reflective Equilibrium

So what is reflective equilibrium? In its most exalted sense, it is the ultimate justification procedure open to some of our beliefs, including our logical beliefs. In a more modest sense, it is a methodology in processes like formalisation, theorification, modelling, etc. These two senses of *RE* are connected and it takes but a small (up and ahead) step from the latter to the former. Both are evident in a celebrated remark of Goodman's, worth reproducing here *in extenso*:

> Principles of deductive inference are justified by their conformity with accepted deductive practice. Their validity depends upon accordance with the particular deductive inferences we actually make and sanction. If a rule yields inacceptable inferences, we drop it as invalid. Justification of general rules thus derives from judgments rejecting or accepting particular deductive inferences. This looks flagrantly circular. I have said that deductive inferences are justified by their conformity to valid general rules, and that general rules are justified by their conformity to valid inferences. But this circle is a virtuous one. The point is that rules and particular inferences alike are justified by being brought into agreement with each other. *A rule is amended if it yields an inference we are*

> *unwilling to accept; an inference is rejected if it violates a rule we are unwilling to amend.* [...] [I]n the agreement achieved lies the only justification needed for either. (1955, 63–64)

Much of what I have to say will target *RE qua* methodology. This is because I take it that whatever problems beset it in this quality, also affect its status as a state that justifies a body of beliefs: *RE* is supposed to generate an eponymous doxastic state in which one's logical beliefs are justified. But if the process does not warrant the cogency of its outcomes, then what value can there be to either? A state of *RE* may be seen as one where no further developments of one's theories is possible because there are no more apparent problems to resolve.[3] Yet the same situation could ensue as an effect of lack of curiosity, of having a deficit of imagination, or low epistemic standards. This kind of epistemic "tranquillity" is a non-specific symptom. Insofar at is has any value, this is due to the inherent virtues of the process that lead to it.

So what is this methodology? Goodman's original description refers only to inferences, principles of inference and the relation between them. But we may well suppose that articulating this relation involves a few more ingredients. So, expanding a bit on the original schematic proposal, we can easily get a *prima facie* plausible story that goes along the following lines: One starts with a body of inchoate, perhaps practical or intuitive, knowledge of a certain domain—for instance, that associated with the dispositions to infer manifested in the daily ratiocinative practice, or even that obtained by a modicum of reflection on the practice. That is, one starts with the knowledge expressed in pre- or quasi-theoretical claims like "this argument is valid", "that doesn't follow", or perhaps even "valid arguments are truth-preserving", etc. Call this "*1-knowledge*."[4]

This body of pre-theoretical knowledge is apt for further regimentation, precisification and expansion—by fine-tuning the conceptual apparatus behind it, by discovering novel, perhaps more abstract or more general, relations between its objects, by forming new hypotheses, proving general statements,

3 This is a somewhat implausible contention, as it is not clear how, for instance, the effort to achieve a *simpler* theory could be massaged into the simple picture of *RE*. But let us grant it for the sake of the argument.

4 I do not wish to attach any precise philosophical sense to the word "knowledge." Instead, it is to be taken in the intuitive sense. To the extent that it is explicit knowledge, it consists of both statements (factive, prescriptive, normative, etc.) and the conceptual apparatus (predicates, relations, etc.) underlying them. However, I am not assuming that this knowledge must be explicit; it can well be, at least partly, knowledge-how.

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etc. Thus, one moves from the knowledge that a particular item is an argument to a general account of what arguments are, from the belief that valid arguments preserve truth to beliefs like "valid deductive arguments preserve designated value on Tarskian models," etc. Call (all) this "*2-knowledge*."

The development and refinement of *2-knowledge*—or, in one word, *theorification*—proceeds and is kept in check by balancing it against *1-knowledge*. Theoretical pronouncements are measured against the pre-theoretical knowledge that inspired them in the first place. For instance, a rather bad putative definition of *argument* as "speech in which, out of two given things, a third follows" is suitably modified upon realising that many (things that are usually called) arguments have more or less than two premises (given things) and may well derive a conclusion (third thing) that is, in fact, identical to (one of) the premise(s).

At the same time, *1-knowledge* is, at least potentially, modifiable in light of *2-knowledge*. For instance, it may be that *1-knowledge* does not provide for a distinction between inductive and deductive arguments (though maybe it could), whereas *2-knowledge* does. This theoretical distinction may inform *1-knowledge* and we may see hosts of savvy informal reasoners resorting to it in everyday contexts. Or it may be that pre-theoretically we are disposed to infer in accordance with a certain form of argument but, in virtue of general principles of validity developed as part of *2-knowledge*, we come to see that this is not the case (cf. *infra*, the discussion of the ω-rule for an illustration of this case.)

Our logical theories and, with them, logical knowledge, are obtained and justified as a result of this trade-off between pre-theoretical and theoretical beliefs.[5]

2 Formalisation and the Formation of Logics

Goodmanian reflective equilibrium seems to presuppose a non-conventionalist view of logic. At any rate, it is easier to grasp the problems of *RE* if we assume, without loss of generality, such a view. Recall Carnap's famous *principle of tolerance*:

> In logic there are no morals. Everyone is at liberty to build his own logic, i.e. his own form of language, as he wishes. All that is required of him is that, if he wishes to discuss it, he must

5 For a more detailed discussion of the method see the opinionated survey in Cath (2016).

> state his methods clearly, and give syntactical rules instead of philosophical arguments. (1937, sec.17)

For Carnap, the standard for the success of logics is not the extent to which they "correspond" to natural language, the medium of human reasoning, but rather their usefulness relative to the purposes for which they were designed.

Not so for the view that will provide the background for the present discussion. On it, the relation between natural language and the logical formalism must go beyond the latter's usefulness in analysing the former. For specificity's sake, let our underlying view of logic be that it is obtained via a process of *formalisation*, understood as "a kind of extraction [...] of logical form" out of natural language (Peregrin and Svoboda 2016, 4)—see also Peregrin and Svoboda (2013, 2017).[6]

The image suggested by *RE* is readily seen to fit some scenarios of "formalisation" which are marked by but two parameters:

1. An informal argument like (arg): "Socrates is mortal because all men are mortal."
2. A target logical system (e.g. first-order logic) or perhaps merely a target logical syntax (e.g. Fregean syntax, by which I mean the sort of syntax that explicitly features sentential operators and construes atomic declarative sentences as having function-argument from, as opposed to, say, subject-predicate form).[7]

Suppose now that we go about formalising (arg) in the Fregean syntax — our target (tar). We already know its syncategoremata: expressions like "all", "some", the (grammatical) conjunctions "and", "or", "if ... then," etc. We also know, by and large, how to deal with them in (tar). All in all, we could arrive at the following schematic rendering of (arg):

$$\frac{\forall x M x}{M s}$$

of which we make sense via a key that says that "M" stands for *mortal*, "x" is a variable ranging over the extension of "man", and "s" an individual constant, standing for *Socrates*.

6 For an alternative account of formalisation, see Brun (2014). For a monographic analysis of the many problems raised by this deceptively simple concept, see Brun (2004).

7 This is not inconsistent with the Peregrin-Svoboda view of formalisation, as the "target" need not be thought of as being antecedently available. It can be just as well be "extracted" in the process of formalisation.

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It's no achievement to see that this is a suboptimal—indeed, plainly wrong—formalisation of (arg). For one thing, "All men are mortal" was rendered formally rather dumbly. For instance, *man* and *mortal* were placed in distinct grammatical categories. Not only is this unpleasantly non-uniform, but it also obscures the predicate status of *man*. We would do better to render this premise as "$\forall x(Wx \to Mx)$", with "W" standing for *man* and x ranging over a (generic) class of objects. (Note that this is already a good step away from the"surface" grammar of English.) So we get an improved rendering of (arg), namely:

$$\frac{\forall x(Wx \to Mx)}{Ms}$$

the validity of which we check in (tar).[8] Obviously, it is not.

Does this mean that the conclusion of (arg) does not follow logically from the premise? Well, yes, it does mean that; still, we wouldn't want to say that "Socrates is mortal" may be false when "All men are mortal" is true. In this sense, we would not want to revise our commitment to (arg). We figure out that we need another premise, "Socrates is a man," in order to validate both (arg) and its formalisation.

And so on and so forth: I am not particularly bent on boring the reader with logical trivia. The salient point is that all this happens within the confines of a more or less precise target formalism. At this level, of *formalisation*, it is quite plausible to see our endeavours as governed by *RE*.

The *formation of logics*, to appropriate a term used by Peregrin and Svoboda (2016, 2017), is, as it were, the next level of formalisation-qua-extraction. One obtains a logic by making explicit (cf. Brandom 1994) *and* bringing together into a coherent ensemble the principles governing informal reasoning. No matter how generous our notion of formalisation is, this is no *mere* formalisation, as a few examples will show.

Consider first the case of a working mathematician who believes, in the first instance, that the ω-rule:

$$\frac{P(0) \qquad P(1) \qquad \ldots \qquad P(n) \qquad \ldots}{\forall x(x \in \mathbb{N} \to Px)}$$

is *logically* valid.

8 Actually, since (tar) is rather imprecise, the validity check would have to be performed in a logic based on the Fregean syntax or, at the very least, in a fragment of such a logic that contains enough information about $\to$, $\forall$, and the horizontal "inference" line that ended up rendering "because."

Subsequently, and in light of various *2-knowledge* beliefs—inference rules are finitary, logic is topic-neutral, "natural number" does not express a logical property, logicism fails because of Russell's paradox, etc.—she changes her mind and decides not only that the ω-rule is not part of logic, but also that its syntactic structure, and in particular its infinite number of premises, make it not an inference rule at all.[9]

Take now Peano's axiom of induction. Its natural formulation involves quantification over properties:

$$\forall P(P(0) \wedge \forall n(P(n) \rightarrow P(n+1)) \rightarrow \forall n P(n))$$

For various (theoretical) reasons, this kind of formalisation was thought best to be avoided and first-order logic, in which the quantifiers range only over individuals, became the norm (for more on this, see Eklund 1996). The demise of second order formalisms has little to do with what goes on in natural language, where (apparent) quantification over properties is certainly present. It was and, to the extent that the controversy is alive, it still is a matter of deploying heady theoretical considerations.[10] Languages may carry logics inside them, but it is still up to the logicians to decide what to bring to the surface and how.

A third example will also illustrate the fact that, in many cases, the practice is not at all coherent and it cannot light our way in a simple fashion. Take the following rules governing a truth predicate T:

$$\frac{A}{T\langle A\rangle}\ T\text{-I} \qquad\qquad \frac{T\langle A\rangle}{A}\ T\text{-E}$$

They seem innocuous enough. But add some equally innocuous reasoning principles and pick the sentence named by $\langle A\rangle$ so that it is "This sentence is false" and all hell breaks loose, i.e. any sentence follows from any sentence.[11] Deciding how to handle these issues significantly exceeds what can be reasonably characterised as a process of formalisation.

Thus, *in practice* the formation of logics is a rough-going process of theorification responsible to the pre-formal practice, informed by it and, allegedly

9 This example may also serve to illustrate the modification of *1-knowledge* in virtue of *2-knowledge* discussed at the end of the previous section.

10 Famously, Quine rejected second-order logic as set theory "in sheep's clothes" (1970, 66). But the same logic was forcefully defended by S. Shapiro (1991).

11 For more on this, see below, section 4.

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at least, placed under its control to a certain extent. The process goes beyond simple formalisation and is not at all unproblematic.

RE is meant to guide us on the righteous path of smoothing out these asperities and forming a justified logic, by debunking whatever tensions may arise between *1-* and *2-knowledge*. Can it really do this? I think not and in the next three sections, I will explore three cases of current logical debates, consideration of which will explain why I am sceptical about the promises of *RE*.

3 Case Study no.1: Multiple Conclusions

Orthodox logical theorising (Dummett 1991; Steinberger 2010) teaches that an argument has one or more premises and only one conclusion. In this it is faithful to the practice, insofar as it appears that natural language arguments have but one conclusion. At the same time, inferences of the form:

$$\frac{\neg\neg A}{A}\ \text{DNE}$$

are generally accepted in the daily ratiocinative practice. That is, one tends to accept inferences by *double negation elimination* (DNE).

As it turns out, these pre-theoretical commitments stand in an uneasy tension, albeit one that needs a rather sophisticated background theory to surface fully. This background theory is a version of logical inferentialism, better known as *proof-theoretic semantics* (Prawitz 1965, 1974; Schröder-Heister 2018; Francez 2015), whose roots can be traced back to Gentzen (1935). Proof-theoretic semantics theorists hold that the meaning of the logical operators is determined by the primitive rules of inference that govern how sentences in which they feature as principal operators are, respectively, introduced and eliminated from proofs. These two kinds of rules for an operator must match; to put it in jargon: they must be in *harmony* (Dummett 1991). If harmony does not obtain, then the operator is illegitimate and so is the inferential behaviour it sanctions. Moreover, the test for the "match" between the introduction and elimination rules is syntactic in nature. There must be a syntactically assessable property the obtaining of which witnesses the harmonious character of the pairing.[12]

12 This is why proof-theoretic semantics is salient for spotting the aforementioned tension: It requires meaning explanations to proceed in terms of syntactical properties against the background of the

DNE is obviously an elimination rule for negation. The corresponding introduction rule is the (intuitionistic) *reductio ad absurdum*:

$$\dfrac{\begin{array}{c}[A]_j \\ \vdots \\ \neg A\end{array}}{\neg A}\ \text{iRAA}, j$$

It turns out that these two rules cannot be harmonised *if* arguments (and the formal proofs representing them) are single-conclusion. A familiar, if bitterly contested, account of harmony has it that a set of introductions and eliminations for a logical constant is harmonious only if its addition to a proof system is conservative (Dummett 1991).[13] That is, to the extent that the addition generates new valid arguments, then these must involve the novel vocabulary. Famously, Peirce's law

$$((A \to B) \to A) \to A$$

despite containing only one logical operator, the conditional, is not provable in intuitionistic logic. *A fortiori*, it is not provable using only the rules for the conditional. However, once one adds DNE to intuitionistic logic—thus ensuring that negation behaves classically — there is a proof of it. (I leave the construction of the proof as an exercise for the reader.) It follows from this that classical negation is not harmonious. The strongest correct rules for negation are those of intuitionistic logic.

But this holds water only if arguments and the formal proofs representing them are single-conclusion. Only in this case does classical negation yield a nonconservative extension of intuitionistic logic. If multiple conclusions are allowed, classical negation is conservative and hence harmonious. In such

rules used and the structure of the proofs. On truth-conditional approaches to the meaning of the logical terms, the syntax of the proof system matters not at all. The behaviour of the logical operators is determined by their truth conditions and it is plain that, at least if one assumes a bivalent notion of truth, there is no way of making A false when $\neg\neg A$ is true. That's the end of the story: whether this behaviour is best tracked by a single- or a multiple-conclusion proof system is irrelevant for the validity of DNE.

13 Not much hinges on this contested account of harmony. It features here because it is the best known. For a defence of it, see Dicher (2016); for criticism, see Read (2000). For a more recent proposal see Gratzl and Orlandelli (2017).

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systems there are proofs of Peirce's law in the implicational fragment alone:

$$\dfrac{\dfrac{\dfrac{\dfrac{[A]_1}{A,B}\,\text{Weakening}}{A,A\to B}\,{\to}\text{I},\,1 \qquad [(A\to B)\to A]_2}{A,A}\,{\to}\text{E}}{((A\to B)\to A)\to A}\,{\to}\text{I},\,2$$

Now let us find our way out of this, guided by *RE*. Assume that our background theory, i.e. the commitment to inferentialism and the account of harmony as conservativeness, is sacrosanct.[14]

The first thing to notice is that the tension we ought to resolve is not between the pre-formal practice and our theoretical commitments. Rather, it is a tension within the practice—albeit one that comes to the fore only against the background of a commitment to a proof-theoretic account of the meaning of the logical vocabulary.[15] It seems that in order to even be able to "reflect equilibristically" on the matter, one must antecedently form some reasonably justified theoretical beliefs about validity, the structure of proofs, etc. In other words, one needs (some theory in order) to *generate* a tension between *1-knowledge* and *2-knowledge*.[16]

On the flip side, this picture suggests that revisions that put in accord the practice with the theory—against the background of its more abstract pronouncements—are somehow inescapable. Alas, it seems to me that it also leads to the demise of *RE* as a *significant* methodological constraint in logical theorising: If we agree that any theory will mutilate in some way some aspects of the practice to which we would otherwise wish to remain faithful, then it follows that any and all resolutions of conflicts must, ultimately, do violence

14 To be sure, this is a contentious assumption. I will say a bit more by way of motivating it in footnote 16.

15 For characterisations of *RE* involving the appeal to a background theory, see Brun (2004, 2013, 2014) and the references therein. Notice that Brun's "background theories" may be more encompassing than those described here.

16 But why would anyone do that? Why not outrightly modify the background theory so that there is no conflict? Presumably, that background theory, including its tension generating aspects, is not embraced idiosyncratically. One clings to it because it explains better other aspects of the practice one is theorising about. It is, in other words, the best theory one has thus far about the target practice. Besides, it is not a stretch to expect that modifications to the background theory will generate other tensions, pertaining perhaps to other parts of the practice. Indeed, it would be foolishly optimistic to expect otherwise.

to the practice or, which amounts to the same thing, to *1-knowledge*. Note that the assumption made is not at all surprising, given that theorification presupposes a great deal of systematisation. In the particular scenario at hand and, consequently, in all scenarios relevantly analogous to it, it is indeed unavoidable, since the practice itself is less than coherent.

The moral of the story is that logical *facts*, as discernible in the vernacular ratiocinative practice, are fragile.[17] They are bound to succumb to the pressures exerted by needs peculiar to theorification or to its perceived benefits. Resolving conflicts is not so much a matter of finding some equilibrium between the practice and the theory, as it is a matter of finding a convenient excuse to obliterate the inconvenient aspects of the practice.

This may appear to blatantly contradict another problem raised with respect to *RE* by Woods (2019). Woods, following Wright (1986), accuses the procedure of suffering irremediably of the problem of "too many degrees of freedom." That is, it leaves open too many areas for revision, mainly with respect to what I have termed here the "background theory." In particular, even the beliefs that brought about the conflict may be subject to revisions. I believe that the contradiction is merely apparent. I've blocked that possibility and kept the background theory unchangeable precisely in order to avoid the degrees of freedom problem *because* I believe that Woods' diagnosis is correct in the absence of that assumption. Now we see that even with it *RE* fares less than stellarly.

One may argue that this does not go against *RE*, which does not require that the resolution of the conflicts be balanced, or "just", etc. All that *RE* requires is that we resolve the tensions between the practice and the theory, even if, as I have claimed, this will systematically ensue in the theory gaining the upper hand. But then it seems that *RE*, as a methodological requirement, amounts to little more than the injunction to pay *some* attention to the domain one is theorising about. This, of course, is a piece of eminently reasonable advice. It is also about as useful in guiding our investigations of that domain as the prophecies of the oracle of Delphi would be in planning one's future.

This, then, is the first complaint that I have against the thesis that *RE* is a meaningful guide to the formation of logics: that "real" equilibrium matters little for it, and that the process of achieving what we may call "internal" equilibrium, is heavily rigged in favour of theoretical considerations.

17 This is abundantly illustrated by the actual solutions to the problem of multiple conclusions; see Dicher (2020).

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4 Case Study no.2: Which Logic is This?

I have already mentioned classical logic. Despite its many merits, few logicians expect classical logic to perform well in the presence of of paradox-generating vocabulary like vague predicates or transparent truth. But are they right in thinking this?

Contrary to these common beliefs, an impressive case has been put forward by Cobreros et al. (2012, 2013) on behalf of classical logic being able to handle the aforementioned troublesome vocabulary without degenerating into a trivial consequence relation (see also Ripley 2012, 2013). To be sure, this is classical logic in a particular and rather special guise—special enough to give it a name of its own: "*ST*", pronounced "strict-tolerant." Let us see us how classical logic and *ST* handle the paradoxes and in what sense the latter is classical.

Our starting point is Gentzen's sequent calculus for classical logic, *LK* (1935). Recall that this contains the Cut rule:

$$\frac{X : Y, A \qquad A, X : Y}{X : Y}$$

Now if one were to add e.g. the *T*-rules from above to *LK*, then the system would become trivial: any conclusion would follow from any premisses. To see this, let λ be a sentence such that $\lambda \equiv_{df} \neg T\langle\lambda\rangle$. Thus λ is the (strengthened) *Liar*: "This sentence is not true."[18]

Then we can derive the empty sequent:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{T\langle\lambda\rangle : T\langle\lambda\rangle}\,\text{Id}}{\neg T\langle\lambda\rangle : \neg T\langle\lambda\rangle}\,\neg\text{-L, }\neg\text{-R}}{\lambda : \lambda}\,\text{df}}{T\langle\lambda\rangle : \lambda}\,T\text{-L}}{: \neg T\langle\lambda\rangle, \lambda}\,\neg\text{-L}}{: \lambda}\,\text{df, Contraction} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{T\langle\lambda\rangle : T\langle\lambda\rangle}\,\text{Id}}{\neg T\langle\lambda\rangle : \neg T\langle\lambda\rangle}\,\neg\text{-L, }\neg\text{-R}}{\lambda : \lambda}\,\text{df}}{\lambda : T\langle\lambda\rangle}\,T\text{-R}}{\neg T\langle\lambda\rangle, \lambda :}\,\neg\text{-R}}{\lambda :}\,\text{df, Contraction}$$

$$\dfrac{\qquad\qquad}{:}\,\text{Cut}$$

from which in turn $A : B$ follows for any A, B via Weakening.

Gentzen (1935) proved that Cut is eliminable from *LK* in the sense that any derivable *LK*-sequent is derivable without using Cut; hence *LK* and its cut-less

18 The truth predicate is essential for expressing λ, though it is not the only required ingredient. The name forming operator $\langle\ldots\rangle$ is equally important. For more technical details about this setup, including the matter of how to render λ expressible, see Ripley (2012).

variant, LK^-, are equivalent in that they derive the same sequents. Since in the above proof Cut is essential for deriving the troublesome empty sequent, we have two proof systems that, although equivalent in the absence of the truth predicate, behave differently when extended with the rules governing it.

LK^- can be used to formalise *ST*,[19] which has the same valid sequents as classical logic but allows for non-trivial and conservative extensions with the sort of vocabulary that generates troubles classically. Semantically, its consequence relation can be characterised by the strong Kleene valuations (Kleene 1952), given below for conjunction, disjunction and negation, when A follows from some premises (bundled in the set) X iff, whenever each of the statements in X has the value 1, the conclusion A has a value in $\{1, 1/2\}$:[20]

$\wedge$	1	1/2	0
1	1	1/2	0
1/2	1/2	1/2	0
0	0	0	0

$\vee$	1	1/2	0
1	1	1	1
1/2	1	1/2	1/2
0	1	1/2	0

$\neg$	
1	0
1/2	1/2
0	1

This brings about a wealth of questions of paramount importance for logical theorising:

- Is *ST* truly the same logic as classical logic or are they different logics? And, if the latter, in what may their difference consist of?
- Is transitivity, as encapsulated by Cut, an essential property of a logic or is it something that we can dispense with?
- And, for that matter, just what (kind of) properties are Cut and similar, sequent-to-sequent, structures?

One thing that seems plain in light of the above discussion is that, if in deciding what logic we are dealing with we keep track only of provable sequents (over the usual language of classical logic), then there is no way to spot the difference between *ST* and classical logic. Is there any (good) reason to so identify logics?

Indeed there is. Sequents are usually construed as *inferences* or claims that the formula(e) on the right-hand side of the symbol ":" *follow from* the

19 Or rather LK^- together with the inverses of the operational rules, see Dicher and Paoli (2021).

20 This interpretation of *LK* goes back to Girard (1976). Note also that, usually, the consequence relation of *ST* is taken to be multiple-conclusion: a set of conclusions follows from a set of premises whenever all the premises are 1 and at least one of the conclusions has a value in $\{1, 1/2\}$.

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formula(e) on the left-hand side of that same symbol. Thus *ST* and classical logic have the same logically valid inferences.

But is this enough when it comes to unequivocally determining the identity of the logic expressed by a formal proof system?[21] The case of *ST* seems to suggest otherwise. One place where the difference between classical logic and *ST* comes to the fore is in the sequent-to-sequent rules they validate. *ST* loses Cut and many other classically valid *sequent-to-sequent inferences* or *metainferences* as they have become known in the literature (Barrio, Rosenblatt, and Tajer 2015; Barrio, Pailos, and Szmuc 2021). Indeed, it has been proved (Barrio, Rosenblatt, and Tajer 2015; Dicher and Paoli 2019) that while the valid sequents of *ST* determine classical logic, its valid metainferences determine the logic of paradox, *LP* (cf. Priest 1979).

The *ST*-theorists are well aware and unperturbed by this fact. For them, these metainferences, or rather the rules they generate, are mere "closure principles" which a consequence relation may or may not obey (cf. Cobreros et al. 2013). Alas, whether or not this is the correct way to look at Cut and other metainferences is a disputed matter. It certainly isn't the only one. For instance, Dicher and Paoli (2021) have argued that a logic is actually an equivalence class determined in a suitable way by those metainferences that are valid in the following sense: any valuation that satisfies the premise sequents also satisfies the conclusion sequents.[22] From this perspective, *ST* is not classical logic, but rather *LP*.

So much for *ST* and its properties; now let us return to *RE*. Suppose that at the end of a careful process of formalising various natural language arguments we end up with the class of classically valid sequents as a codification of the class of valid inferences. Have we thereby also settled the matter of whether we have formalised classical or strict-tolerant logic? I believe that we have not and that we have *formed* our logic while somehow failing to form an accurate idea of which logic it is. For that, we need to answer a few more questions: What are we to make of the loss of Cut and other metainferences in *ST*? Or of the fact that *ST*, unlike classical logic, appears to be somehow ambiguous between two different consequence relations, the classical one and that of

21 This question can be asked with respect to similar, if simpler situations, see e.g. Hjortland (2013), where it is shown how one proof-system can express two different logics. See also Dicher (2020).

22 This is "local" metainferential validity. In contrast, one speaks of global metainferential validity when the universal quantifier is wide scope: for any valuation, if it satisfies the premises, then it satisfies the conclusion.

LP? These are central, albeit very abstract, problems in logical theorising and certainly salient issues in the *formation* of logics.

Is there any hope that *RE* can meaningfully guide us when we set about settling them? At first blush, one may expect that it ought to: after all, the debate is ultimately a debate over the role and status of Cut. The scenario, boiling down to deciding whether a particular (and rather special) metainference rule is valid seems to fit quite well in the Goodmanian framework. But this deceptively simple question quickly spirals out of control, becoming an arcane matter about obscure properties of logical systems and even about how these systems codify consequence relations. It is not just a case of revising, say, our concept of consequence such as to allow non-transitive relations to count as such.

The sort of questions raised by *ST* and its designation as "classical" cannot be answered by following the imperative of reaching an equilibrium between (intuitively acceptable) inferences one is not willing to give up and one's views about which rules of inference ought to be accepted. Even the framing of the problem exceeds the resources available within the *RE* model.

As with problematisation, so with problem-solving.[23] Reaching a *RE* underdetermines the issues at hand. To see this, assume for the sake of the argument that the problem can be meaningfully framed as a typical Goodmanian problem (and also bracket the many details at play in the debate around *ST*).

What is apparent is that something has to go, either the principle of inference codified by Cut or the vocabulary that makes it possible to express Liars, together with its associated inferential resources.[24] Whatever "firm" anchor point the pre-formal practice might provide us, such as, for instance, the almost universal acceptance of transitivity as a property of consequence relations, rather quickly loses its appeal. This inference principle generates inferences we are unwilling to accept, *if* we let it interact with other, equally intuitive, principles such as the *T*-rules. Plainly, *RE* cannot tell us which way to proceed and what to sacrifice—at least because all the inference principles at play have a good pre-theoretical hold on us.

23 This is where the "too many degrees of freedom" problem, already hinted at above creeps upon us.

24 Indeed, other options are possible, but I stick to the limits of the scenario above. Notice also that it is not just liars that are problematic. Vagueness, for instance, can lead to the same problems and be treated in like manner.

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This is not incompatible with it being possible to defend one or another solution. But those solutions and their defences must, of necessity, rely on something more than doing justice to the pre-formal intuitions. Moreover, their virtue simply cannot be that they have balanced our pre-theoretical commitments with our pre-theoretical practice, for this virtue could be boasted by many rival solutions.

5 Case Study no. 3: Paraconsistent Christology and *FDE*

Very recently, JC Beall (2019) took to investigating the so-called *fundamental problem of christology* (cf. Pawl 2016) in light of his favourite logic, *FDE* or *first-degree entailment*. Briefly, the problem is that Patristic theology consecrates the dual nature, divine and human, of Christ. Being divine, Christ is immutable; being human, he is mutable. As a god, Christ is omnipotent; as a human, his powers are limited, etc. Christ, in other words, is possessed of inconsistent attributes. Of him, it is true both that "Christ is *P*" and that "Christ is not *P*," for a good number of essential predicates *P*. Because contradictions are bad in that they do not further the objective of achieving rational knowledge of the object that "embodies" them, this is a problem for christology.

Beall argues that the best solution to this problem is also the simplest: bite the bullet and accept that Christ is a contradictory object. That, however, is not really a bad thing. In particular, he argues, it does not entail that rational theological inquiry about Christ is impossible. Contradictions may be true of Christ, but they are not as *bad* as traditional (Aristotelian, classical, etc.) logicians took them to be. They can be handled by appropriate logics. Thus Beall argues that the proper logic for analytic Christology is the paraconsistent *FDE* (Anderson and Belnap 1975; Belnap 1977).

In its most common guise, *FDE* is a four-valued, truth-functional, and structural logic that recognises, as Beall puts it, a space of logical possibilities that allows a statement to be *true* (= 1), *false* (= 0), *both true and false* (= *b*, a "glut"), and *neither true nor false* (= *n*, a "gap"). The following matrices show how these mappings can be extended to valuations:

$\wedge$	1	*b*	*n*	0
1	1	*b*	*n*	0
b	*b*	*b*	0	0
n	*n*	0	*n*	0
0	0	0	0	0

$\vee$	1	*b*	*n*	0
1	1	1	1	1
b	1	*b*	1	*b*
n	1	1	*n*	*n*
0	1	*b*	*n*	0

$\neg$	
1	0
b	*b*
n	*n*
0	1

Both 1 and *b* are designated values and a conclusion *A* follows from some premises *X* if and only if, whenever the premises are at least true, the conclusion too is at least true.[25]

Theological and para-theological considerations aside, I agree with Beall, at least in the following sense: One's best hope of achieving a state of *RE* between the orthodox patristic determinations of Christ and one's logical beliefs is to endorse a paraconsistent logic. *Ceteris paribus*, *FDE* will do just marvellously.

But now suppose that one would wish to reject *FDE* on account of being too weak: it does not recognise as valid a great deal many inferences that we have a "natural" propensity to accept.[26] By the lights of *RE*-theorists, this should count against it. But could such criticism be levelled against *FDE* on the basis of *RE* considerations? Alas, it is difficult to see how this could be done. The *FDE* theorist has a very quick way out of this difficulty. All she needs point out is that the incriminated inference is not *logically* valid (after all, it is not *FDE*-valid), although it may be valid within some restricted domain of inquiry, maybe because the predicates of that domain have some special properties. By *FDE* lights, those inferences need not be rejected *simpliciter* though they are rejectable as a matter of logic. While indeed *FDE* is very weak, it can peacefully co-exist with various strictly speaking non-logical strengthenings of it.

So far, this has nothing to do with Christology, paraconsistent or otherwise. But suppose that a *FDE* theorist's *main* reasons to uphold this logic have to do with it cohering with her theological beliefs, in particular with her belief that Christ is an inconsistent object.[27] One trying to dislodge *FDE* as an (all-purpose) logic would be in quite a pickle. It seems clear that one could not move the *FDE* theorist to change her view. Indeed, why would she do so? Not only would this require that she give up a state of *RE*, but it would require her to do so despite having a very handy way of retaining it, i.e. denying the logicality of the *FDE*-invalid inferences while admitting that they are domain-limited valid (or perhaps analytical, etc.). At the limit, such a logician may even claim that *FDE* is too weak for *every* other domain but Christology.

25 *Mutatis mutandis*, the same definition applies to multiple-conclusion formalisations of *FDE*. For sequent calculi for *FDE*, see Beall (2013), L. Shapiro (2017).

26 This task fits well with the main burden that the proponents of sub-classical logics have had to grapple historically: that of giving up as little as possible of the power of classical logic.

27 "Main" as used here is simply meant to signal the importance that our paraconsistent logician ascribes to coherence between their logical theological beliefs.

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This is by no means an irrational claim, despite the seeming exoticism of the preoccupation with the divine nature in this age.[28] And it would certainly help her continue being in the state towards which our theorising must strive, that of *RE*.

There is nothing wrong with this in either the present or in any particular case whatsoever. The problem is that this is a pervasive trend: Setting a state of *RE* as the ultimate justification for our logical beliefs will tend to render weak logics immune to criticism. Quite simply, it seems very unlikely that an *FDE*-opponent of the kind described will ever be in as good a state of (reflective) equilibrium as an *FDE*-champion. The *FDE* theorist can be in equilibrium with respect to their mathematical, logical, theological and in particular Chalcedonian, and whatnot beliefs. And, presumably, a trivialist who believes that there are *no* logically valid arguments, can do even better.

This is a pathological condition to the extent that it means that weaker logics will systematically have a better chance of being justified by *RE*, simply because *RE* is easier to obtain for such a logic. Worse, given the role and purpose of *RE*, there is little incentive to aim for stronger logics.

One may reply that this is not so: A weaker logic means sacrificing — as far as logic is concerned—some inferences which we are generally willing to accept. But both the practice and other logical considerations may press exactly for their acceptance *qua* logically valid. That is true. But to the extent that these considerations are forced upon us by the practice, then, as we have already seen, they are easily brushed aside. The tendency to accept a given inference says nothing as to whether the inference is logically valid, restrictedly logically valid, analytically valid and so on. It is something that needs to be integrated and explained within a bigger theoretical picture. (So we reach again to our old conclusion that (seemingly) logical facts are fragile.) If, on the other hand, the aforementioned considerations are of a theoretical nature, then the justification process itself does not appear to be one whose stake is the successful or coherent integration of pre-theoretical beliefs with theoretical ones. Rather, it appears to be a game of making the best case for one's theoretical conviction. There can be no doubt that doing justice to the "facts" will be part of this process; it is just implausible that it will be the dominant part.

28 By contrast, a logician that would aspire towards coherence between her logical beliefs and the reasoning mistakes she most commonly commits would presumably be acting irrationally.

6 Epilogue

These, then, are the main problems with *RE* as a guide to logical theorising: First, theoretical considerations appear to always be able to undercut whatever tendencies may exist in the pre-formal practice. This means that understood as a methodology, *RE* is too weak because one of the "reflecting" surfaces itself is too weak. Second, I have argued that this methodology underdetermines both the identification of the specific problems one may encounter in "the formation of logics," i.e. problematisation, and the problem-solving process itself. Finally, *RE* systematically favours weaker logics. The weaker a logic is, the easier it will be to bring its prescriptions into harmony with other beliefs we may hold.

Part of the drama of reflective equilibrium is that it appears to fit parts of the (empirical) process of theorification, in particular, formalisation. There is little reason to doubt that the process of theorification starts by working on some raw materials—real inferences, made by real people in the real world. It also seems to me that it is correct to say that the processing of these data is both kept in check by the data and informs them in its turn. This much is inescapable insofar as we take logic to be an applied theory, i.e. our theory of *correct* reasoning (Priest 2006, ch.8).

That, however, does not make *RE* a plausible methodological constraint on, and even less so an appropriate account of the justification of, theorification—not when the chips are down. So, while the Goodmanian image with which we have started is tempting enough, turning it into a successful recipe for logical theorising turns out to be a hopeless job.[29]

At the fringe, reflective equilibrium becomes what the Senate and the consulate were in imperial Rome. One pays lip service to them. One uses them for ritual purposes. Every now and then one looks to them for (very) rough guidance to avoid too extravagant errors. And that's about it. The real power lies with the pretorians: the highly disciplined, highly skilled, and utterly unscrupulous theoretical considerations.

29 I am not alone in reaching this conclusion. See e.g. the previously quoted paper by Woods (2019) and also Wright (1986), S. Shapiro (2000). For recent critical discussions of *RE* in non-logical contexts, see McPherson (2015), Kelly and McGrath (2010). An impressive array of objections to *RE* is surveyed and critically discussed in Cath (2016).

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7 Postscript

Despite having reached the end of the story, the paper must go, because an anonymous referee asked the most important question to which I did *not* wish to answer here: "What are the viable alternatives?".

I stand by my decision not to answer this question here, because I cannot do it justice within the space of this paper. Still, a few words, gesturing towards my favoured answer, may be useful.

Let this be my starting point: I have framed reflective equilibrium as a method embodying a fallibilist epistemology of logic. My criticism of *RE* did not concern the suggestion that logical inquiry is fallible, that we can be wrong in our identification of the "laws of logic," etc. Nor did I challenge the claim that (parts) of the processes of logical theorisation and theorification can be described as proceeding according to a successive series of revisions of the "theory" in light of the "data" and conversely. What I have challenged is the claim that this can be turned into a substantive methodological requirement that would ensue in a justified logical theory.[30] To that extent, I do not wish to endorse fully an apriorist epistemology of logic.

These are the standard (or at least traditional) options in the epistemology of logic. I incline towards a different viewpoint. Thus the answer to the question "What is the best methodology for logical inquiry?" requires a preliminary answer to a deeper question, about how we should think about logic. As for the answer to this last question, Allo (2017, 546) puts it best:

> [I]t makes sense to think of logic as a kind of cognitive technology: a tool or set of tools used to reason more efficiently. The proposal to see logic as conceptual technology extends the scope of this picture, and emphasises that all the core notions that logical systems give a formal account of (like validity, consistency, possibility, and perhaps even meaning) should be understood as artefacts

30 It seems to me that this is not completely false even of a priori methodologies for logic. It is one thing to argue, however (im)plausibly, that the validity of *modus ponens* is known *a priori* by dint of knowing the meaning of *if … then*. (The disjunction between plausible and implausible, suggested by e.g. McGee's (1985) alleged counterexample to *modus ponens* should by itself give us pause.) It is a rather different thing to argue that the same is true of, e.g. vacuous discharges of assumptions, which are essential for ensuring a monotonic behaviour of the conditional. Likewise, it is one thing to argue that transitivity is an analytic note of the concept of "logical consequence" and quite another to decide whether this is to be captured at the inferential or metainferential level.

> that shape deductive reasoning practices rather than as neutral descriptions or codifications of pre-existing inferential practices.

So the referee's question "What are the viable alternatives?" has a simple but hardly informative answer: Whatever methodology best serves the imperative of developing the best cognitive technology that logic can be. What that actually means is a matter for further thinking.*

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