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# Considerations on Logical Consequence and Natural Language

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In a recent article, “Logical Consequence and Natural Language,” Michael Glanzberg (2015) claims that there is no relation of logical consequence in natural language. The present paper counters that claim. I shall discuss Glanzberg’s arguments and show why they don’t hold. I further show how Glanzberg’s claims may be used to rather support the existence of logical consequence in natural language.

Contemporary logic is studied using the tools of formal languages that have been developed during the past two centuries. Logicians often approach natural language with some apprehension: natural language is complex and messy, studied fragment by fragment by a variety of methods that hardly seem to provide any sense of unity. This is by contrast to formal languages, that are neat, manageable and simple (to the extent that the logician devises them to be). Is there even a logic in natural language? If we are to move beyond first impressions, we should make precise what we mean by this question, and specifically, what we mean by “logical consequence,” “natural language” and logical consequence being “in” natural language.

In a recent paper, “Logical Consequence and Natural Language,” Michael Glanzberg (2015) confronts this issue head-on. While the literature is not short of remarks on the question of the relation between logic and natural language, Glanzberg’s important contribution is a paper-long discussion of what may be meant by the question and an extensively argued response. It is therefore worthwhile to consider the details of Glanzberg’s arguments, and thus further the discussion on this fundamental topic. This contribution is thus dedicated to discussing Glanzberg’s stance, and to criticising the arguments he puts forward. Now, if we are to present a critique of Glanzberg’s argumentation, it would be most fruitful to do so on Glanzberg’s terms: on his understanding of the question of logic in natural language. However, we shall be critical not only of his response to the question at hand but also of the particular

constraints that he imposes which lead him to his response. Taking up some basic assumptions from Glanzberg, we are led to very different conclusions than his. Before I delve into Glanzberg's reasoning, let me start with a broader introduction to help us orient ourselves in the discussion.

Here, together with Glanzberg, we shall treat natural language as a natural phenomenon—as the object of study of empirical linguistics. Logical consequence will be taken to be a relation between sets of sentences (constituting premises) and sentences (serving as conclusions) in the relevant language. This relation holds if the conclusion necessarily follows from the premises by virtue of the form of the sentences. We shall elaborate on this condition later on, but for now let us note that formal systems studied by logicians can be taken to be displaying, or modelling logical consequence in natural language. Our understanding of what this relation might be will be tied to the options exhibited by formal systems. That the relation of logical consequence is *in* natural language will be explained to mean that the appropriate formal systems for logic serve as good models for a phenomenon in natural language.

The formal systems we shall refer to are the products of a tradition starting with Frege's *Begriffsschrift*, which set as a primary aim to provide a methodology for the sciences. At the base of this tradition we have first and second order predicate logic—and as the aims varied and developed through the twentieth century, so did the formal systems that were used. Examples of other partakers in this traditional project, who upheld the same primary aim, are Tarski, Carnap and Quine. The virtues they sought in logical systems had to do with their uses in scientific reasoning—whether in deductive sciences (Tarski's primary target) or beyond (as we can see in Carnap and Quine).

Formal systems have as their first and foremost virtues rigour and mathematical precision. Further virtues, which can be attributed to the basic systems (first order logic and possibly some of its extensions), would include simplicity and restrictiveness. If, for example, we consider Frege's foundational project, we see that the epistemological motivations of placing arithmetic on a secure ground lead invariably to a restrictive stance towards logic.<sup>1</sup> Other members of the traditional project held a similar attitude, each in their own way.<sup>2</sup>

The formal systems devised by Frege and his successors have found their way to a variety of applications and uses, where different emphases called for different virtues. Relevant to our discussion is the *linguistic project*, which

1 Frege helped himself to second order logic, which, following Glanzberg, will be considered as restrictive for the purpose of this paper.

2 I discuss the traditional project in length in Sagi (2020, 2021).

we can see developing from the midst of the twentieth century onwards, where formal systems of logic are used in the study of natural language (as in Chomsky 1957; Davidson 1967, 1970; Davidson and Harman 1972; Montague 1974).

The traditional project has distinctive normative aspects, at least insofar as it is methodological. The linguistic project, by contrast, is wholly descriptive. Natural language, disregarded by members of the traditional project as inadequate for scientific research, here becomes the main focus. Natural language, as the subject matter of linguistic theory, is treated like any other natural phenomenon. The formal systems devised in the traditional project become useful tools for the formal study of natural language syntax and semantics. Rather than a medium for formulating scientific theories, now the formal systems become mathematical models for the study of natural language.

It is very clear, however, that the restrictive systems of the traditional project are much too coarse, and are inadequate in capturing a wide array of natural language phenomena. First or second order predicate logic may be suitable for foundational purposes, but it is hardly a good fit for linguistic study. This mismatch is where the suspicion arises that logic and natural language lie on very different grounds. Glanzberg's arguments are essentially based on the observation that standard predicate logic fails to be a good fit for the study of natural language, and he therefore concludes that natural language, on certain assumptions, does not have a genuine consequence relation.

Before moving on, I'd like to pause on the relation between the formal systems provided by a linguistic theory and the phenomenon which is the subject matter of investigation. Cook (2002) gives us a way of assessing this relation. Cook (2002, 234) presents us with three rough options. We can take the formal system to be a description of natural language and its logical properties: on this view, every aspect of the formalism corresponds (at least roughly) to a feature of the phenomenon being formalized. On the other end of the spectrum, we can view the formalism as completely instrumental: it might help us in predictions on the phenomenon at hand, but the details of formalism provide us with no insight or explanation of the inner-workings of the phenomenon. These two options lay a spectrum of possible views, where somewhere in the middle we can find the view of logic-as-modelling. In this view (see also Shapiro 1998), the formalism serves as a mathematical model of the phenomenon at hand. Some aspects or elements of the model correspond to features of the phenomenon (these are *representors* in Shapiro's terminology), and others (*artefacts*, in Shapiro's terminology) do not: they

help keep the model simple and easy to handle. It seems that the extremes of the spectrum are either impractical or unhelpful, and that a reasonable approach would be to aim for some place in the middle.

In the present context, when we ask whether there is a logical consequence relation in natural language, one way to approach the issue would be to see whether formal systems that satisfy basic conditions we would expect from systems for logic are good models for some phenomenon in natural language. I shall claim that Glanzberg himself provides the basis for the position that formal systems of logic are indeed models of natural language phenomena.

The plan of the paper is as follows. In section 1, I present the thesis of logic in natural language as understood through Glanzberg's terms, and I articulate the basic assumptions and observations that are essential for Glanzberg's reasoning. Glanzberg presents three arguments against the thesis of logic in natural language. I review and counter these arguments, each in turn, in section 2–section 4. Besides the negative arguments, Glanzberg also presents a positive proposal of how a logical consequence relation can be obtained by modifying natural language. In section 5, I shall argue that the process described by Glanzberg is that of modelling, and it thus serves to rather substantiate the thesis that there is a logic in natural language.

## 1 Making Sense of the Question: Glanzberg's Analysis

Glanzberg argues that natural language does not have a logical consequence relation. More specifically, he argues that when logic is understood in the appropriate restrictive way, the following thesis is false:

*The logic in natural language thesis:* a natural language, as a structure with a syntax and a semantics, thereby determines a logical consequence relation. (2015, 75)

Glanzberg explains that *logic* can be understood either restrictively or permissively. The more restrictive the logic, the less inferences it accepts as valid. Basically, standard, classical first or second logic are of the restrictive sort by Glanzberg's lights, and the variety of "non-standard" and "non-classical" logics include the permissive sort (2015, 78). According to Glanzberg, the arguments he presents show that natural language does not determine a restrictive logical consequence relation, and strongly suggest that it also does not determine a permissive logical consequence relation.

We shall deal with Glanzberg's arguments in the following sections. First, however, let us lay out the claims that serve as the basis for Glanzberg's arguments.

First, we note that Glanzberg analyses logical consequence as a *necessary* and *formal* relation (2015, 76). It is necessary in the sense that a valid argument is an argument where truth is preserved from premises to conclusion over all relevant possibilities. It is formal in the sense that it holds by virtue of the forms of the sentences involved. There is, of course, much more to say, but this should suffice at present.

Now, Glanzberg (2015, 79) crucially assumes a model-theoretic account of logical consequence, and that such an account is most likely to lead to a logical consequence relation in natural language. I do not object to this assumption, but it would be helpful to see what it is based on. Glanzberg's model-theoretic approach builds on three observations. The first one is that post-Tarskian model-theoretic consequence is necessary and formal as required (Glanzberg 2015, 77). Secondly, model-theoretic consequence appears to be a good explication of logical consequence—understood as necessary and formal (notwithstanding well-known criticisms like Etchemendy 1990).

The third observation which bases the model-theoretic approach is that in the study of natural language, we find a family of related notions, among which are implications and entailments. According to Glanzberg, *implication* is a wide notion, covering relations that are either logical or of looser connections, including those based on defeasible reasoning. Within the category of implications, we have the narrow notion of logical consequence, that which aligns with the restrictive view of logic (see Glanzberg (2015, 80); apparently even though logical consequence is a subspecies of implication, it is not really a relation in natural language—more on this in what follows). And included in implications we have *entailment*, which is understood as a truth-conditional connection:  $p$  entails  $q$  if the truth conditions of  $p$  are included in the truth conditions of  $q$  (Glanzberg 2015, 80). Entailments include analytic connections, such as “Max is a bachelor, therefore Max is unmarried,” and they may include also “metaphysical” connections, such as “ $x$  is water, therefore  $x$  is  $H_2O$ .” That is if truth conditions are metaphysically possible worlds, and one accepts the Kripke-Putnam views of natural kind terms (Glanzberg 2015, 80).

In sum, we have on the one hand model-theoretic consequence, which fits the analysis of the notion of logical consequence. On the other hand, we have relations in natural language that come structurally close to, and even include as a subset the relation of logical consequence thus conceived.

Glanzberg's argumentation from this point onwards serves to draw a divide between model-theoretic consequence and the broader relations we find in natural language.

Another crucial assumption made by Glanzberg is that the way to determine whether there is a relation of logical consequence in natural language is through looking at current practices in linguistics, and more specifically, those of contemporary natural language semantics. To a certain extent, I find this assumption justified: linguistics is the science that studies natural language. If the state of the art in linguistics either enforces or undermines the existence of a certain phenomenon in natural language, we should certainly take that into primary consideration. Glanzberg, however, seems to draw more from contemporary semantic theory, and we shall review this issue in due course.

Glanzberg presents three arguments to support his conclusion: the first leans on the assumptions we spelled out above, and the other two have additional assumptions which will be brought up in our further discussion. In the following sections, I shall give an outline of the arguments and present my criticism. The outcome will be that Glanzberg's arguments are not as strong as they aim to be, and do not give sufficient basis to refute the logic in natural language thesis.

## 2 The Argument From Absolute Semantics

The first and main argument Glanzberg puts forward is the *argument from absolute semantics*. It is the most general of the three arguments, and it concerns the use of model theory in natural language semantics. The gist of the argument is that natural language semantics is *absolute*, and in fact does not use the range of models that model theory offers.

One of the basic ideas, adopted from Lepore, is that model theory defines only *relative* truth conditions. It gives us the notion of truth in a model. It says, for instance, whether the sentence "Snow is white" is true in some model. Semantic theory, if apt, should give conditions of truth *simpliciter*, i.e. tell us when "Snow is white" is true. Davidsonian *absolute* statements of truth conditions tell you that the sentence "Snow is white" is true if and only if snow is white, which, according to Glanzberg, is what we wanted.

Glanzberg claims that even semantic theories that use model theory, stemming from the Montagovian tradition, are, at bottom, providing absolute semantics. Glanzberg writes:

What is characteristic of most work in the model-theoretic tradition is the assignment of semantic values to all constituents of a sentence, usually by relying on an apparatus of types (cf. Chierchia and McConnell-Ginet 1990; Heim and Kratzer 1998). Thus, we find in model-theoretic semantics clauses such as:<sup>3</sup>

- (1) a.  $\llbracket \text{Ann} \rrbracket = \text{Ann}$
- b.  $\llbracket \text{smokes} \rrbracket = \lambda x \in D_e. x \text{ smokes}$

[...] [These clauses] provide absolute statements of facts about truth and reference [...] We see that the value of “Ann” is Ann, not relative to any model. (2015, 89)

Semantics of natural language, according to Glanzberg, is the study of speakers’ linguistic competence, and more specifically, of knowledge of meaning. Arguably, truth conditions are what a speaker knows when they understand a sentence. The relevant study must then be directed at the absolute values presented in the clauses above. By contrast, Glanzberg explains, in order to understand the logical properties of a sentence, we look at the values of the sentence across a range of models. But since semantics of natural language is absolute, it is blind to what happens across any non-trivial range of models (2015, 91). To sum: whether natural language has a logical consequence relation will be determined by whether current semantic theory appeals to a non-trivial range of models in explaining speakers’ competence. Since it doesn’t, natural language, according to the argument from absolute semantics, does not have a logical consequence relation. Later on in the article, Glanzberg concedes that a range of models is explicitly appealed to in the study of determiners, but, he explains, at this point semantic theory goes beyond its proper terrain. We shall reach this point in due course.

Is natural language semantics really absolute? Here are some considerations to the contrary. Note that while the semantic value of “smokes” is a function which determines for every object in the specified domain whether it smokes, semantics does not tell us what this function is—what its values are. Indeed, all that semantics gives us is the *condition* for obtaining the value 1 from this function. And so, all we have, in extensional semantics, are truth conditions

<sup>3</sup> Glanzberg explains: “In common notation,  $\llbracket \alpha \rrbracket$  is the semantic value of  $\alpha$ . I write  $\lambda x \in D_e. \phi(x)$  for the function from the domain  $D_e$  of individuals to the domain of values of sentences (usually truth values)” (2015, 89).

of a sentence such as “Ann smokes” rather than an absolute truth value. Heim and Kratzer explain that the semanticist cannot, and also should not, provide the function in extension: “We do not know of every existing individual whether or not (s)he smokes. And that is certainly not what we have to know in order to know the meaning of ‘smoke’” (1998, 21). Reference is not what a speaker knows. While the meaning of an expression determines a reference, what the speaker knows does not pick out the reference. This indeterminacy makes room for a range models.

Thus, despite the form of the clauses above, when we look at the practice of natural language semantics, we do find a range of models. In Zimmermann (1999) it is claimed that a range of models is a part of natural language semantics, and that it reflects linguists’ ignorance. Linguists can’t point out the extension of every expression in natural language. If they could, it would be determined by natural language semantics whether there are white ravens or whether Ann smokes, merely by giving the extensions of “white,” “ravens,” “Ann” and “smokes.” If we are interested only in one model, then the relation between extensions is completely determined.<sup>4</sup> Now one might insist that natural language semantics does require an absolute semantics, and that the range of models is a byproduct of less than ideal theorising, not indicative of any real phenomenon in natural language. But note that the ignorance of linguists is not (at least not always) expected to be overcome, as we see from the quote of Heim and Kratzer. It is not part of linguistic competence whether Ann smokes—or on which possible worlds Ann smokes. It is not only the linguist’s ignorance that a range of models may signify, but also that of competent speakers themselves.

Indeed, another recent article by Glanzberg suggests that the explanatory power of semantic theory is limited where absolute items such as (3a-b) are involved, and that such clauses contain pointers to other cognitive faculties. “[S]emantics, narrowly construed as part of our linguistic competence, is only a partial determinant of content” (2014, 259). We need further conceptual resources to fully determine the extension of every expression in a language.

Now, while I take the above considerations to undercut the absoluteness of natural language semantics, I submit that the argument from absolute semantics fails even if we accept that natural language semantics is absolute.

<sup>4</sup> If we use possible world semantics, the extensions of expressions may vary from world to world, but then the modal profile of the term’s extensions would have to be known if a single model is used. Moreover, in such semantics there’s usually an “actual world” singled out which would have to match the actual extensions of terms.

Let us review Glanzberg's reasoning: Natural language semantics should indicate whether natural language has a genuine logical consequence relation; the subject matter of natural language semantics is linguistic competence; a key aspect of linguistic competence is knowledge of truth conditions; truth conditions do not require a range of models; a genuine logical consequence relation requires a range of models; therefore, there is no genuine logical consequence relation in natural language. It seems to me that all that this reasoning establishes is that the study of truth conditions in natural language is not identical to the study of logical consequence in natural language, a mark of the difference is that one uses a range of models and the other does not. Glanzberg begs the question when he looks for logical consequence in natural language by looking at a discipline which he defines through its subject matter, which is not logical consequence.

In Glanzberg's words: "semantics of natural language—the study of speakers' semantic competence—cannot look at [a range of models] and still capture what speakers understand" (2015, 91). The present claim would thus be that while a range of models would not give you all that is understood by speakers, it is what it takes to give a logical consequence relation in natural language.

This is not to claim that natural language semantics is the wrong place to look for logical consequence. We are still left with the possibility that there is a sub-phenomenon that can be identified as a logical consequence relation. Now, entailment, which is a phenomenon studied by natural language semantics, is a wider category than logical consequence according to Glanzberg. So if it is the putative narrower phenomenon of logical consequence in natural language that we were to study, we would need to adjust our toolkit accordingly. We would need to appeal to a range of models. Acknowledging this is not to dispute that natural language semantics, as the study of truth conditions and entailment, is absolute—it is merely to distinguish another, related (indeed—narrower) phenomenon.

We should add that looking at a range of models does not require more information on words' extensions beyond what natural language semantics gives us. Defining "Ann" as a singular term whose extension varies between models requires less information than giving its absolute extension. And so, natural language semantics contains all the information that is needed for the range of models involved. We may thus still agree with Glanzberg that natural language semantics is the place where we should look for a relation of logical consequence in natural language, if such exists—and we may even find it there. If it is the range of all entailments with which a native speaker is

competent, then they are *inter alia* competent with the subset of entailments that are logical. If a competent speaker knows truth conditions of sentences most generally, then they also have the specific knowledge that is required for merely the logical entailments, as the latter is contained in the former. This point is also relevant to Glanzberg's *argument from lexical entailments*, to which we turn next.

At this point, however, we might be accused of overlooking an important piece of information required for moving to a range of models: we need to be able to distinguish between the logical and the nonlogical vocabulary. That is because, when moving to a range of models, we let the extensions of nonlogical expressions vary (according to their semantic category), while the extensions of the nonlogical vocabulary remain fixed. It might then be claimed that the distinction between logical and nonlogical expressions is not provided by natural language semantics, and that it extends the phenomena that can be found in natural language. Indeed, this is Glanzberg's argument from logical constants—which we address in section 4.

### 3 The Argument From Lexical Entailments

Next, Glanzberg presents the *argument from lexical entailments*. While natural language semantics does not require a range of models, it does look at the range of possibilities that account for truth conditions. The nearest thing to logical consequence that we find, then—according to Glanzberg—are entailment relations. However, entailment, as we have seen, is presumably much broader than a restrictive notion of logical consequence, since it includes analytic and metaphysical implications. Furthermore, entailments seem to completely forgo formality—many entailments depend on lexical components of sentences. Here enters an additional assumption made by Glanzberg, concerning formality. What determines the forms of sentences are *logical constants*, and logical consequence holds in virtue of their properties (Glanzberg 2015, 77). The meanings of the nonlogical vocabulary are abstracted away. Indeed, as we've mentioned, the standard model-theoretic conception of logical consequence has us completely fix the meanings of some of the vocabulary (the logical terms) and maximally vary, in line with semantic category, the meanings of the rest of the vocabulary (the nonlogical terms). On this common conception, if an argument is accepted as valid, and the validity of an argument depends on the specific meaning of an expression

appearing in it, that expression must be treated as logical, and its meaning should be fixed across models.

The logical vocabulary, on this conception, constitutes a small, distinguished subset of the whole vocabulary. In standard first order logic we include the truth-functional connectives and the universal and existential quantifiers. Glanzberg mentions that logical constants normally have certain criteria imposed on them, such as topic-neutrality or permutation or isomorphism invariance. We shall mention criteria for logical vocabulary in the next section. Here, we may note that a choice of logical vocabulary determines a consequence relation. Moreover, the stricter we are with respect to logicality of expressions, the more restrictive is the consequence relation that results.

Now, entailment is a phenomenon in natural language, and, as implicated by Glanzberg, it is the most reasonable candidate for being natural language's logical consequence relation. Entailments, however, according to Glanzberg, depend on the meanings of nonlogical expressions.

Glanzberg provides the following examples of entailments to prove his point:

- (1) a. We loaded the truck with hay.  
ENTAILS  
We loaded hay on the truck.
- b. We loaded hay on the truck.  
DOES NOT ENTAIL  
We loaded the truck with hay.
- (2) John cut the bread.  
ENTAILS  
The bread was cut with an instrument.

[...]

These entailments are fixed by aspects of the meanings of words like “load” and “cut”. (2015, 93–94)

The words “load” and “cut” are noncontroversial examples of *nonlogical* expressions—in a reasonably restrictive model-theoretic consequence relation they would not be fixed. One can presumably, on a permissive view of logic, study the logic of words like “load” and “cut,” and so consider them as logical constants. But, according to Glanzberg, lexical entailments permeate language too far for us to have anything like a strict separation between logical and nonlogical constants. Practically every word would have to be considered

as logical—that is since practically every word has lexical entailments that depend on its meaning. Furthermore, the lexical items above obviously do not fulfil accepted criteria for logicity.

The argument from lexical entailments may be objected to on two counts: one regarding the assumption that all lexical entailments as the examples above would have to be included in natural language's logical consequence relation, and another regarding the assumed conception of formality. As for the first: recall that according to Glanzberg, logical consequence is a narrower relation than that of entailment, and it is included in it. Above, we have examples of members of the difference between entailment and logical consequence. Entailments that are also logically valid would depend for their validity only on the meanings of the distinguished logical vocabulary (whatever that may be). What prevents us from taking these special entailments and marking them members of the logical consequence relation of natural language? Logical consequence, according to Glanzberg, is not a totally alien relation to natural language. Indeed, it is a subset of an accepted relation in natural language. What is to prevent us from marking it as its own phenomenon, in natural language?

Here is one way to respond. Take an accepted natural phenomenon, say that of organic compounds, studied in organic chemistry. Among the organic compounds, we have those liked by Sara the chemist. We thus have a subset of a chemical phenomenon that can hardly be considered as its own chemical phenomenon. So, while the items exemplifying the phenomenon fall squarely within the subject matter of the relevant science, what distinguishes them—being liked by Sara—is not a feature relevant to the science. Do we have the same case with logical consequence? Is its distinguishing feature a matter of the scientific study of language, and in particular, of natural language semantics?

In the previous section, I claimed that if logical consequence is a sub-phenomenon of entailment, then surely it calls for a proper adjustment of the toolkit for studying it, including a range of models rather than an absolute semantics. The argument from absolute semantics does not refute the existence of this sub-phenomenon. However, now we confront an intriguing question, for which I don't claim to have a definite answer: which distinctions are relevant to the subject matter of natural language, and which are not? We could aim at a principled definition of the subject matter involved to arbitrate the matter, or we might aim at more social considerations, and see whether work of researchers in the relevant field employ such distinctions. Observing

the discipline of natural language, Glanzberg claims that while entailment is marked as a self-standing studied phenomenon, logical consequence is not. Now, on the assumption of formality, the matter turns on whether the distinction between logical and nonlogical expressions is relevant, whether it is one that can mark a phenomenon in natural language. This is the issue tackled in Glanzberg's *argument from logical constants*, with which we deal in the next section. There I shall object to Glanzberg's exclusion of the distinction between logical and nonlogical terms from the realm of natural language.

I've mention another line of objection to the argument from lexical entailments, having to do with the assumption of formality. Admittedly, formality is a widely accepted a condition on logical consequence (Beall, Restall, and Sagi 2019).<sup>5</sup> Glanzberg can certainly not be blamed for assuming the common conception of formality on which to base his conclusion against the existence of a restrictive logical consequence relation in natural language. However, for the sake of the more general discussion, I'd like to mention an alternative approach to logical consequence, which may still accept the examples of entailments above as logical validities without trivializing formality. Note that in order to capture the above entailments, all that is needed is some restriction on the meaning of the words "load" and "cut" or their meanings' relations with the meanings of other words. Indeed, one need not completely fix the extension of these words in order to obtain these entailments. In previous work, I have proposed a model-theoretic framework for logical consequence where there is no strict division of the vocabulary into logical and nonlogical: terms are fixed in various manners and to various degrees using *semantic constraints*—restrictions on admissible interpretations of terms (Sagi 2014). As we have clauses in standard first order logic fixing the interpretation of the logical vocabulary, we may have clauses only restricting the interpretations of terms without fixing them completely.<sup>6</sup> Without pursuing this line any fur-

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5 Notwithstanding some exceptions, *debunkers* by the terminology of MacFarlane (2015), by whom logical consequence is not defined as formal, even if logicians avail themselves with formal tools to study this relation (see Read 1994; and other references in MacFarlane 2015).

6 These clauses may remind of *meaning postulates*, as in Carnap (1952); Montague (1974). An important difference is that while for Carnap and Montague the clauses for the logical vocabulary are treated as basic, onto which meaning postulates are added, in the framework of semantic constraints all kinds of constraints (whether those completely fixing the meaning of a term or those akin to meaning postulates, only restricting meanings of terms) are treated on a par, and they determine the forms of sentences—and thus the formality of the obtained consequence relation is upheld.

ther,<sup>7</sup> we may take note that there are alternative approaches to formality, on some of which the logical validity of the arguments above does not entail that “load” and “cut” need to be fixed as logical terms. On such approaches, it may very well turn out that entailment itself is a formal relation, and constitutes the logical consequence of natural language.

#### 4 The Argument From Logical Constants

Finally, Glanzberg presents the *argument from logical constants*. We have mentioned the criterion for logical terms of invariance under isomorphisms. The idea is the following. Logical terms are general, and they do not make distinctions between elements of the domain. Therefore, their extension remains constant under permutations of the domain: switching between members of the domain cannot entail a difference in the extension of a logical term. For example, the extension of the first-order existential quantifier is taken to be the set of all nonempty subsets of the domain, and so it is invariant under isomorphisms: no permutation of the domain can transform a nonempty set into an empty one, or vice versa. Similarly, logical terms are indifferent to switching between members of the domain and members of other domains, and are therefore invariant under isomorphisms.

We shall leave technicalities aside to the extent that we can.<sup>8</sup> Here it would suffice to acknowledge the role of the criterion of invariance under isomorphisms in a current conception of logical consequence. This criterion has been defended extensively in the literature (Sher 1991, 1996) or at least accepted as a necessary condition for logicity. By this criterion, the standard quantifiers and identity relation of first order logic are logical, but in addition, so are the variety of generalized quantifiers, such as *Most* and *There are infinitely many*. Thus, one might think that the grammatical category of determiners in natural language includes logical constants that would salvage formality and the feasibility of a logical consequence relation in natural language. For instance, let us observe the semantic clause for the determiner “most” (cf. Glanzberg 2015, 98):

- a. Local:  $\llbracket \text{most} \rrbracket_M = \{ \langle A, B \rangle \subseteq \mathcal{P}(M)^2 : |A \cap B| > |A \setminus B| \}$
- b. Global: function from  $M$  to  $\llbracket \text{most} \rrbracket_M$

7 I intend to explore applications of the framework of semantic constraints to natural language semantics in future work.

8 For a detailed survey, see Westerståhl (1989).

The semantic clause has a *local*, absolute, part, which, given a (or rather, “the”) domain, returns pairs of subsets of the domain satisfying the condition. The second part of the clause generalizes over all model domains, making the operator *global*. According to Glanzberg, all that semantic theory requires is the local condition: this condition suffices for accounting for truth conditions of sentences involving “most.” Why then do we have the global extension? Glanzberg (2015, 99) contends that the global condition serves as a useful abstraction, that goes beyond the needs of semantic theory. And so, some properties of determiners can only be captured through their global definition:

- a. CONSERV (local): For every  $A, B \subseteq M$ ,  $Q_M(A, B) \Leftrightarrow Q_M(A, B \cap A)$
- b. UNIV (global): For each  $M$  and  $A, B \subseteq M$ ,  $Q_M(A, B) \Leftrightarrow Q_A(A, A \cap B)$

It is claimed that natural language determiners satisfy these conditions, and thus they in fact express restricted quantification. The global property UNIV is generally stronger (see also Westerståhl 1985), and it requires a range of models. Glanzberg explains that at this point we depart from natural language semantics:

In looking at this sort of global property, we are not simply spelling out the semantics of a language. Rather, we are abstracting away from the semantics proper—the specification of contributions to truth conditions—to look at a more abstract property of an expression. (2015, 100)

On Glanzberg’s approach, what decides whether some phenomenon is part of natural language is its relevance to the determination of truth conditions. Glanzberg raises the option of still viewing isomorphism invariant determiners as logical constants, since they have a property accepted by many as a distinguishing feature of logical constants. But according to Glanzberg, what is distinctive of such expressions is that they are amenable to extensive mathematical treatment—a property held by non isomorphism invariant terms as well. “So,” Glanzberg concludes, “natural language will not hand us a category of logical constants identified by having a certain sort of mathematically specifiable semantics.” And “Is there anything else about language—anything about its grammar, semantics, etc.—that would distinguish the logical constants from other expressions? No” (2015, 101). By more permissive lights, not limited to isomorphism invariance, we might accept the greater class of functional categories as including the logical expressions of a language,

which is distinguished grammatically. But if we remain within the restrictive viewpoint, we see, according to Glanzberg, that logicity is not part of natural language.

To the argument from logical constants too I object on two counts. Natural language contains expressions that satisfy accepted criteria for logicity, such as “most” and “more.” Specifically, these expressions are invariant under isomorphisms. Glanzberg claims that this criterion does not latch onto a natural phenomenon, and the category of logical constants is not recognized by natural language. Now, while isomorphism invariance might not delineate a standard grammatical category, it does, arguably, spell out a property that distinguishes some expressions from others. An expression that is invariant under isomorphisms arguably does not distinguish the identity of individuals (Sher 1991, 43). This is a property that, in this view of logicity, makes these terms logical. If there is a phenomenon such as logical consequence in natural language, and logical consequence is analysed as requiring a distinguished set of logical terms, then this distinction would be made in its theory. So if invariance under isomorphisms is accepted as the distinguishing criterion, and there are expressions in natural language that satisfy it, what else do we need in order to say that there is a category of logical expressions in natural language?

Now, echoing the discussion from section 3, one might not be satisfied with this response. Perhaps, still, this distinction is artificial, and logical consequence is thus forced on natural language. It is unclear what makes a distinction external or artificial, but we can claim that in this case, indeed, one can defend the distinction and argue further against the putative artificiality. Moreover, whether or not natural language distinguishes between logical and non-logical terms is not a settled matter in the literature. Glanzberg takes the work in Westerståhl (1985) to go beyond natural language semantics, perhaps because of its highly abstract, mathematical nature. But we can find the relevant distinction in more empirically-oriented, mainstream natural language semantics. In some recent studies in linguistics it has been proposed that language does indeed separate between logical and other entailments. Gajewski (2002) argues for a category of sentences that are *L*-analytic—true or false in virtue of form—as a special case of ungrammaticality, based on speakers’ intuitions. Presumably, his account can be extended to include entailments. Fox (2000) and Fox and Hackl (2006) argue that the cognitive system contains a deductive system in which sentences are evaluated and ruled out if they can be proven to be contradictory. Fox’s characterization of the deductive

system, as well as Gajewski's characterization of the L-analytic sentences employ a distinction between logical and non-logical words, where logical words correspond roughly to the logical terms in standard first order logic. Chierchia builds on these ideas to develop a full-fledged theory of the relation between logic and grammar. According to Chierchia, it may be that logic and grammar are distinct computational systems, yet they are interfaced with each other. Logic, in any such case, is a natural phenomenon, and its notions play a central role in grammar (Chierchia 2013). If contemporary semantic theory sets the standard, then there is a basis for distinguishing a class of logical expressions.

## 5 Modelling Logical Consequence in Natural Language

Glanzberg indicates two ways that can lead us to accept a version of the thesis of logical consequence in natural language. One is by considering logical consequence from a more permissive perspective. We shall not discuss this option. The other is by a process of stepping away from semantics proper to obtain a logic. The process is threefold. We first identify the logical vocabulary, by whichever criterion we choose to employ—which (if minimally restrictive) will already at this point take us beyond natural language semantics (according to Glanzberg). Next, we abstract away from the meanings of the nonlogical expressions and allow for a range of domains—and in this way we obtain a range of models that will move us away from absolute semantics. And then, we idealize: natural language is full of exceptions and grammatical complications absent in logical systems. The outcome would be much more similar to a consequence relation in a formal language than what we seemed to have started out with. Indeed, Glanzberg contends that the result of this process is a logical consequence relation, and moving away from natural language makes it possible.

Now, let us consider the process Glanzberg describes, that we briefly delineated above. I'd like to argue that this process enforces the stance that there is a relation of logical consequence in natural language, and that through the said process we can model this phenomenon. Recall our discussion in the introduction. When we use a formalism to model a natural phenomenon, it will include representors and artefacts: aspects or elements that will correspond to features of the phenomenon modelled, and those that do not. We invariably idealize and abstract away from many of the features of the phenomenon. Does this mean that what we describe was not really out there, and was made

possible by the process of modelling? Rarely in science does a phenomenon simply jump out at us through a microscope: modelling is part and parcel of the study of complex phenomena. Glanzberg himself relates the process he describes to modelling in science:

Idealization, as it figures here, is a familiar kind of idealization in scientific theorizing that builds idealized models. One way to build idealized models is to remove irrelevant features of some phenomenon, and replace them with uniform or simplified features. A model of a planetary system is such an idealized model: it ignores thermodynamic properties, ignores the presence of comets and asteroids, and treats planets as ideal spheres (cf. [Frigg and Hartmann 2012](#)). When we build a logic from a natural language, I suggest, we do just this. We ignore irrelevant features of grammar, and replace them with uniform and simplified logical categories. (2015, 113f)

Is a planetary system not a natural phenomenon, and part of the subject matter of astronomy? Is it merely a product of modelling, or is it the target phenomenon of a highly abstract model? Inasmuch as the planetary system is a natural phenomenon, and relevantly analogous to logical consequence in natural language, then logical consequence in natural language too is a natural phenomenon.

How could one still question that the process yields a model of logical consequence as a part of natural language? The only stage that can raise doubts is that of identification. Abstraction and idealization are no doubt a part of modelling. The question is whether we are identifying any real phenomenon. If not, then there is nothing that would tie our model to empirical reality. In the arguments from lexical entailment and from logical constants, Glanzberg relies on a certain conception of the formality of logic, that includes the following two assumptions: that a sharp division of the vocabulary into logical and nonlogical is material for the determination of the relation of logical consequence, and that invariance under isomorphisms is a good criterion to be considered in this discussion. So what needs to be identified here is the category of logical constants. Another way to put the question is to ask whether logical constants in our theory are representors or merely artefacts. We have already disputed Glanzberg's arguments against their identification capturing something real. So, given some assumptions accepted by Glanzberg,

we see that the process of identification, abstraction and idealization, rather than bringing into natural language something new, reveals a feature of it by way of modelling. We may conclude this section with the claim that the logic in natural language thesis is still a viable one.

## 6 Conclusion

Let us take stock. The question of logical consequence in natural language is a fundamental one. In order to make any kind of progress, we must explicate the question, and give a clear understanding of what either a positive or a negative response would entail. Michael Glanzberg gives us, besides arguments for a specific response, a basis on which this question can be discussed and understood. The present critique is meant to pick up the discussion, and hopefully move it forward.

We've seen that there are reasons to doubt that natural language semantics is absolute, as claimed by Glanzberg. We've also seen that even if it is absolute, this does preclude the study of phenomena in natural language from appealing to a range of models. We take it on board that the putative phenomenon of logical consequence in natural language would constitute a relation that is included in that of entailment. Now, as we've briefly mentioned, one might find a way to define logical consequence as a formal relation so that it *coincides* with the relation of entailment. Admittedly, that would take a permissive approach to logical consequence by Glanzberg's lights. Alternatively, we might distinguish a subset of entailments as the relation of logical consequence in natural language. While entailments may depend on the meanings of any expressions in the language, logical validities depend only on the logical vocabulary. So in order to distinguish logical consequence as a relation in natural language, we need to identify the logical vocabulary. The logical vocabulary may be characterized by a widely accepted criterion of invariance under isomorphisms.

The question is then whether this feature is one that falls within the purview of natural language. If natural language semantics is the relevant discipline to be studying the putative relation of logical consequence in natural language, the question is whether the distinction between logical and nonlogical terms is relevant to natural language semantics. Logical terms, characterized by isomorphism invariance, are general in that they make no distinction among individuals in a given domain. I see no reason why natural language semantics should not help itself to such a property. Indeed, we've cited linguists who

appeal to this property as an integral part of their work—are they all not studying natural language anymore, when they appeal to this property, but are rather doing something else? This, I would take to be a contentious claim. In sum, the logic in natural language thesis has not been refuted by Glanzberg's arguments.

The thesis of logic in natural language is reinforced when we consider how the relation of logical consequence can be identified and studied through a process of modelling. Glanzberg contends that we can obtain a relation of logical consequence in natural language through a process of identification (of the logical vocabulary), abstraction and idealization. I have suggested that as long that we are identifying something real—as long as our model in the end contains representors of a real phenomenon—what we obtain through the delineated process is a model of a real phenomenon. Specifically, if logical constants in the formalism we use do indeed represent a feature of natural language, then through the formalism we obtain a model of a bona fide linguistic phenomenon.

There is a long-standing sentiment that logic and natural language are disparate entities, and that it is a mistake to associate one with the other. Glanzberg gives substance to this sentiment through meticulous analysis and argumentation. However, I have argued that Glanzberg's approach may very well lead us to *accept* the thesis of logic in natural language. This leaves us with a negative option and with a positive option: either find what it is that may still drive logic and natural language apart that goes beyond Glanzberg's assumptions,<sup>9</sup> or use the tools of natural language semantics or empirical linguistics more generally figure out what the logic of natural language just is.\*

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<sup>9</sup> It seems to me that a characterization of logic as a normative discipline, e.g. along the lines of the traditional-methodological project delineated in the introduction might provide a basis for the claim that there is no logical consequence in natural language.

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